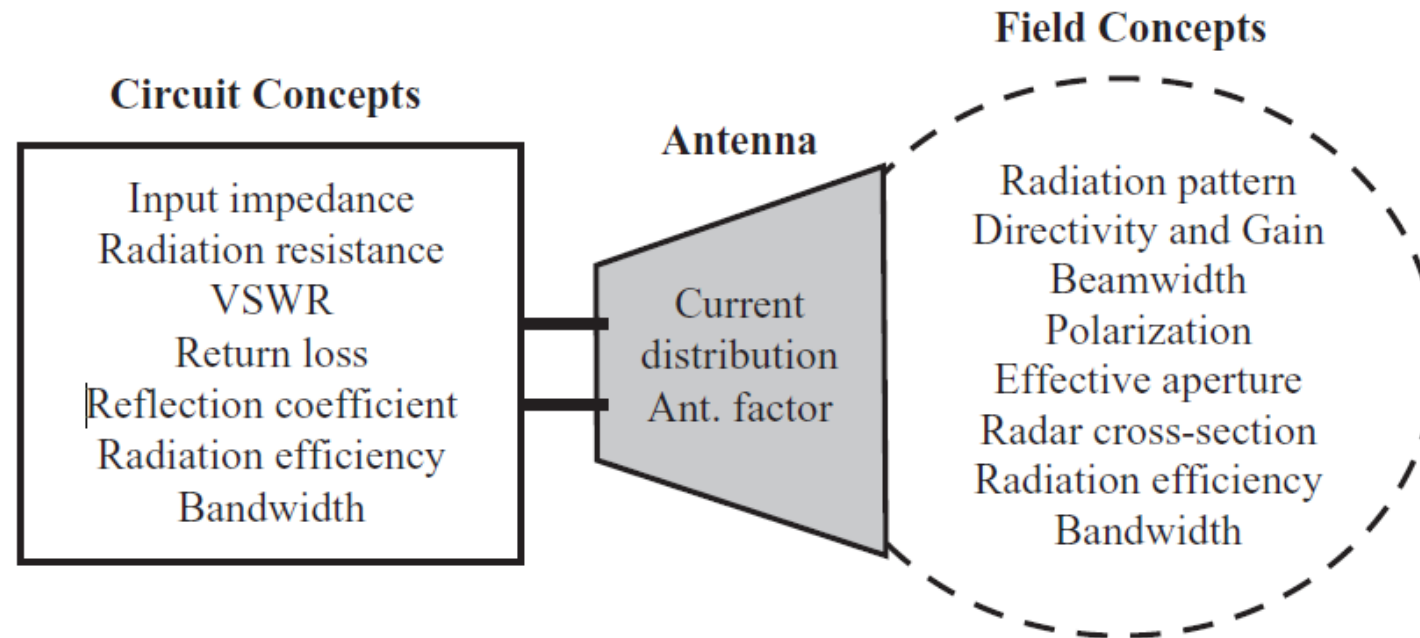# Kablosuz Haberleşme Sistemleri
"Antenler"

Dr. Cahit Karakuş, 2019

How electromagnetic waves bending in the atmosphere?

The most important antenna parameters

Circuit Concepts
Input impedance
Radiation resistance
VSWR
Return loss
Reflection coefficient
Radiation efficiency
Bandwidth

Antenna
Current distribution
Ant. factor

Field Concepts
Radiation pattern
Directivity and Gain
Beamwidth
Polarization
Effective aperture
Radar cross-section
Radiation efficiency
Bandwidth

What is an antenna?

- A usually metallic device (as a rod or wire) is used for radiating or receiving electromagnetic waves. An antenna is a transitional structure between free-space and a guiding structure (Balanis; Antenna Theory).

- An antenna is an electrical conductor or system of conductors
  - **Transmission** - radiates electromagnetic energy into space
  - **Reception** - collects electromagnetic energy from space
- In two-way communication, the same antenna can be used for transmission and reception

History Of Antenna

- 1884, James Clerk Maxwell
  - Calculated the speed electromagnetic waves travel is approximately the speed of light.

  - Visible light forms only a small part of the spectrum of electromagnetic waves.
- 1888, Heinrich Hertz
  - Proved that electricity could be transformed into electromagnetic waves.
  - These waves travel at the speed of light.
- 1896, Guglielmo Marconi
  - Built a wireless telegraph, a spark gap transmitter & receiver
  - On December 12, 1901, accomplished the "Atlantic Leap" from Poldhu, Cornwall, England to Signal Hill, Newfoundland

Antenna functions

- Transmission line
  - Power transport medium - must avoid power reflections, otherwise use matching devices
- Radiator
  - Must radiate efficiently – must be of a size comparable with the half-wavelength
- Resonator
 - Unavoidable - for broadband applications resonances must be attenuated

Antennas

Transmitting Antenna: Any structure designed to efficiently radiate electromagnetic radiation in a preferred direction is called a *transmitting antenna*.

Wires passing an alternating current emit, or *radiate*, electromagnetic energy. The shape and size of the current carrying structure determines how much energy is radiated as well as the direction of radiation.

dipole   loop   slot   patch

helical   horn   reflector

Receiving Antenna: Any structure designed to efficiently receive electromagnetic radiation is called a transmitting antenna

We also know that an electromagnetic field will induce current in a wire. The shape and size of the structure determines how efficiently the field is converted into current, or put another way, determines how well the radiation is captured. The shape and size also determines from which direction the radiation is preferentially captured.
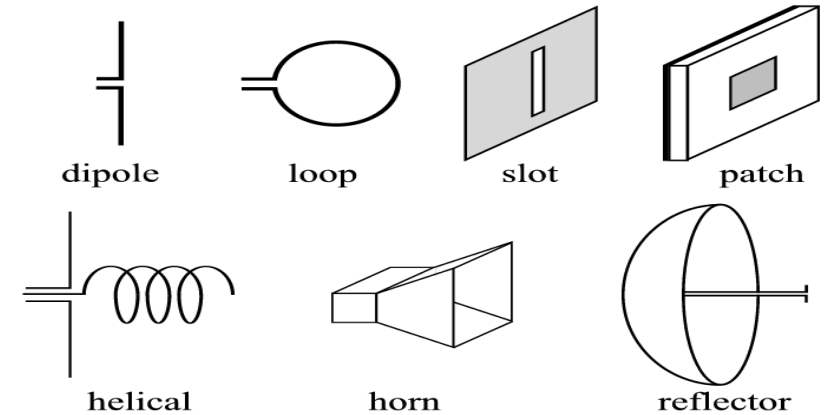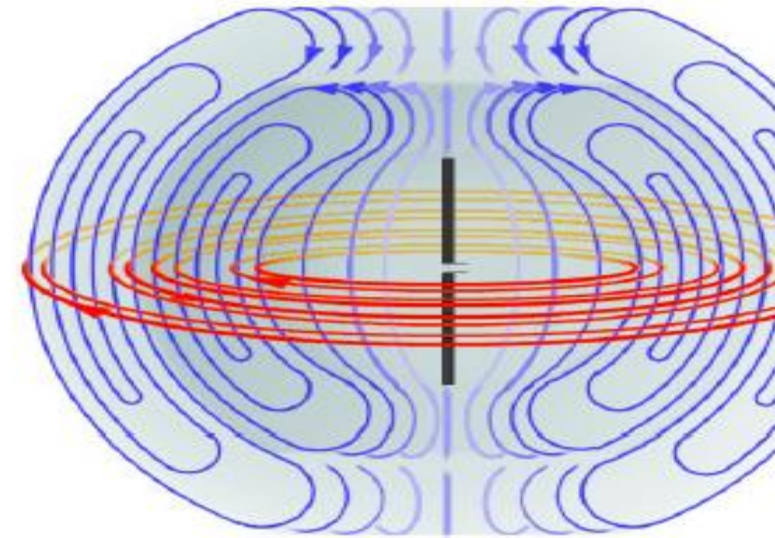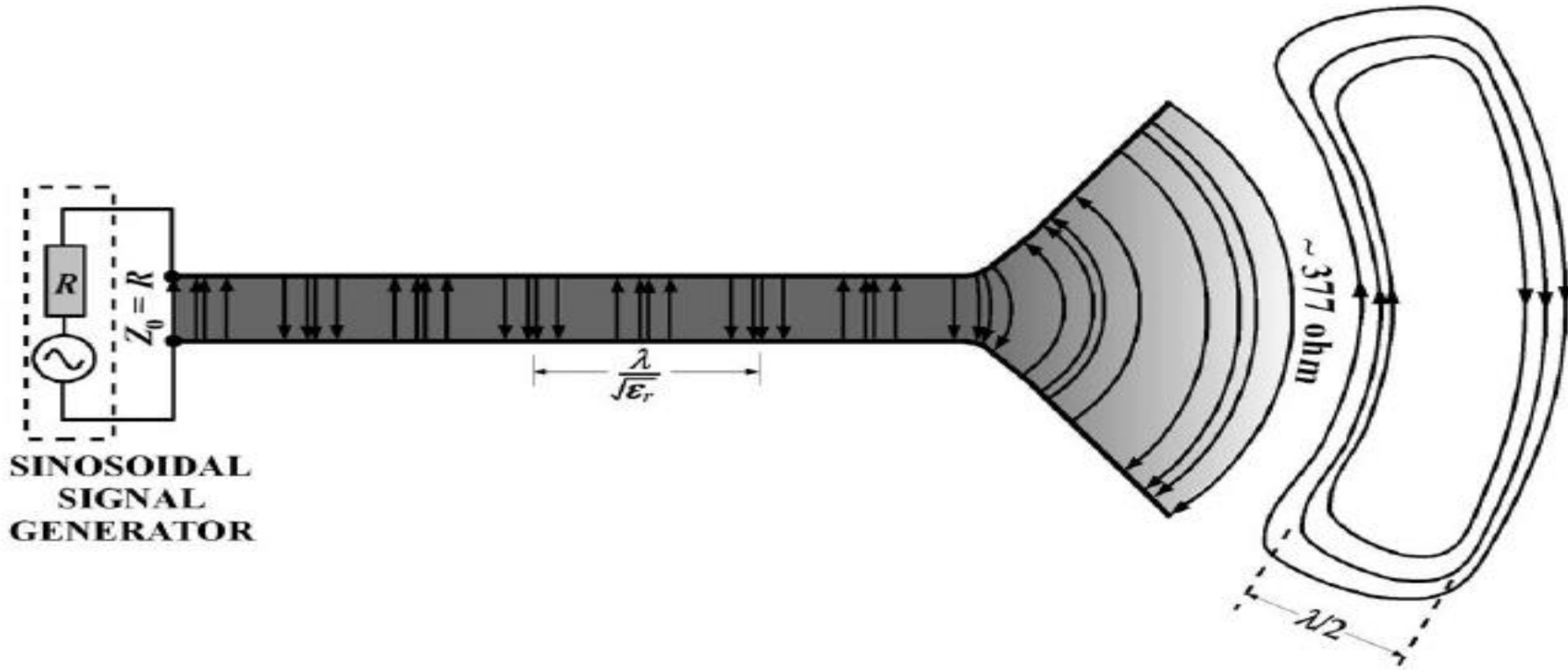
Propagation mode adapter

During both transmission and receive operations the antenna must provide the transition between these two propagation modes.
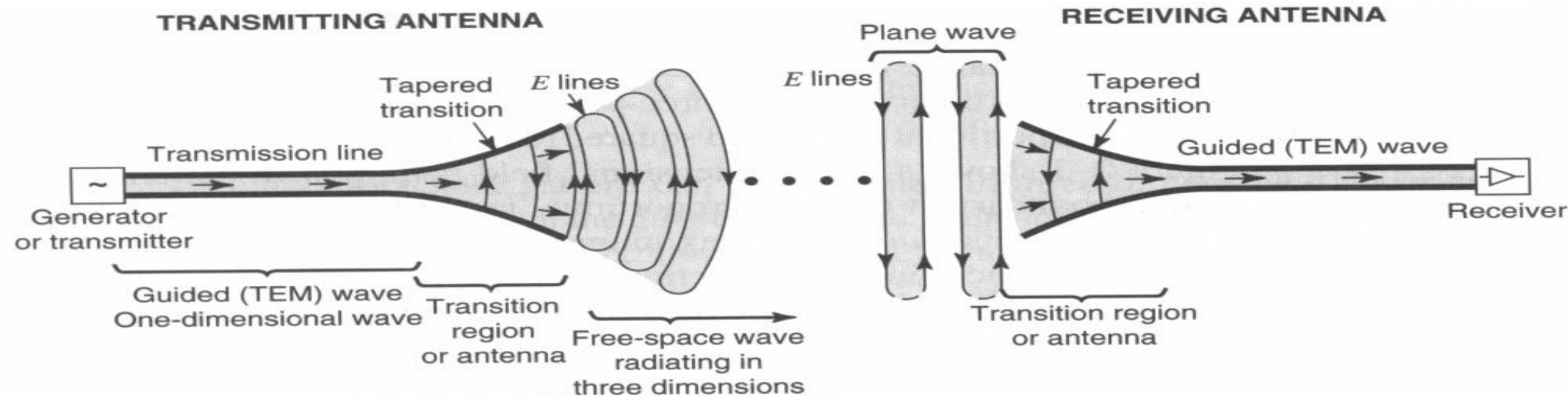
Antenna Radiation and Reception

~377 ohm

$\frac{\lambda}{\sqrt{\varepsilon_r}}$

$\lambda/2$

SINOSOIDAL
SIGNAL
GENERATOR

R

$Z_0 = R$

Dipole radiation fiel
Electric field (bl
Magnetic field (
(picture from wikipe

Due to absence of transmission line conductors, the field lines j
together and an electromagnetic wave is generated with spherical w
front whose source is the signal generator connected at the input enc

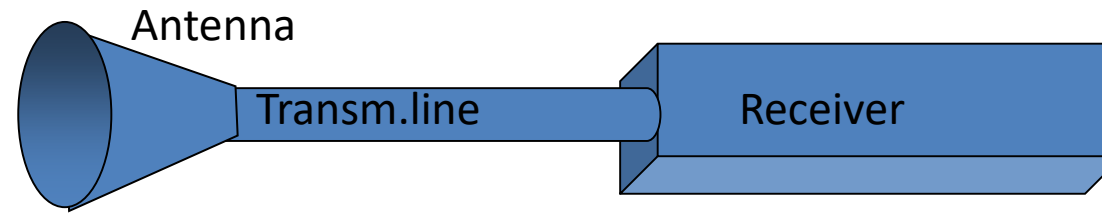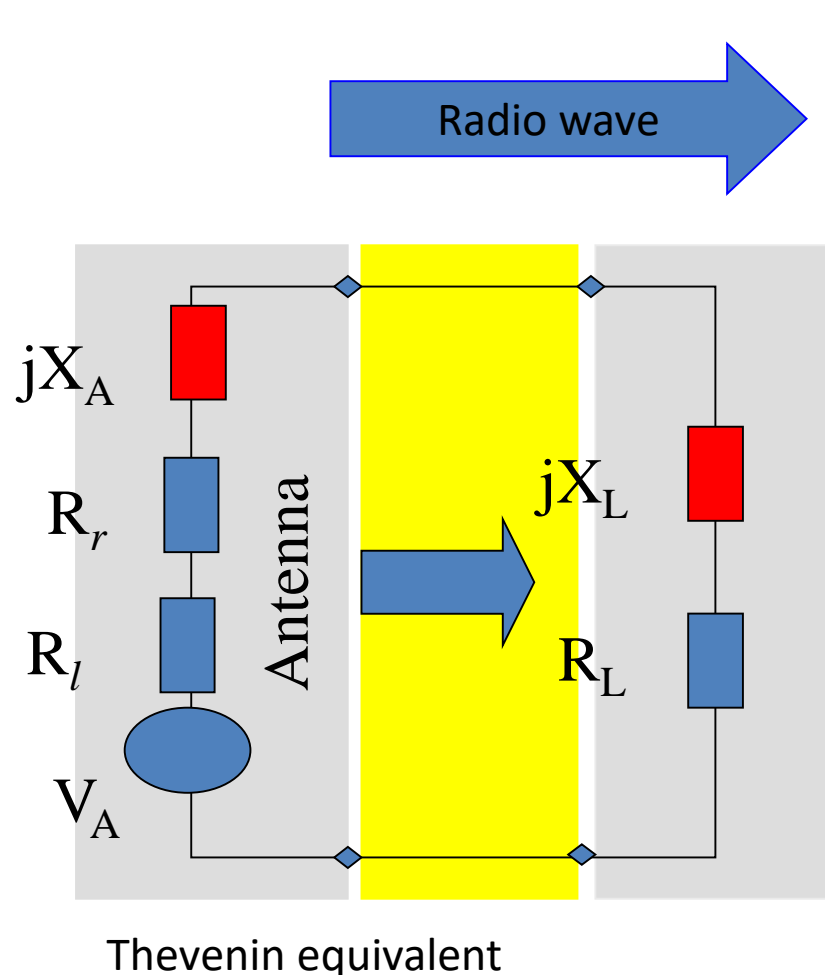# Impedance transformer

Intrinsic impedance of free-space, E/H

$$\eta_o = \sqrt{\mu_o / \varepsilon_o}$$
$$= 120\,\pi$$
$$\cong 376.7\,\Omega$$

Characteristic impedance of transmission line, V/I

A typical value for Z_0 is 50 Ω.

Clearly there is an impedance mismatch that must be addressed by the antenna.

Receiving antenna equivalent circuit

Radio wave →

Antenna

Transm.line — Receiver

jX_A

R_r

R_l

Antenna

V_A

→

jX_L

R_L

Thevenin equivalent

The antenna with the transmission line is represented by an (Thevenin) equivalent generator

The receiver is represented by its input impedance as seen from the antenna terminals (i.e. transformed by the transmission line)

V_A is the (induced by the incident wave) voltage at the antenna terminals determined when the antenna is open circuited

Note: The antenna impedance is the same when the antenna is used to radiate and when it is used to receive energy

Antenna Theory

Theory – wire antenna example

Consider a thin linear antenna of arbitrary length l, with no restrictions on l compared with the wavelength λ. The antenna, shown in Fig. 3.9, is fed at its center with a sinusoidal current distribution,

$$I = I_0 \sin\left[k(l/2 - |z|)\right].$$

The far-zone fields from an infinitesimal dipole of length dz at a distance s are

$$dE_\theta = \frac{jk\eta I}{4\pi s}\, dz\, e^{-jks} \sin\theta_s, \qquad dH_\phi = \frac{1}{\eta} dE_\theta.$$

The far fields of the entire antenna now may be obtained by integrating the fields from all of the Hertzian dipoles making up the antenna:
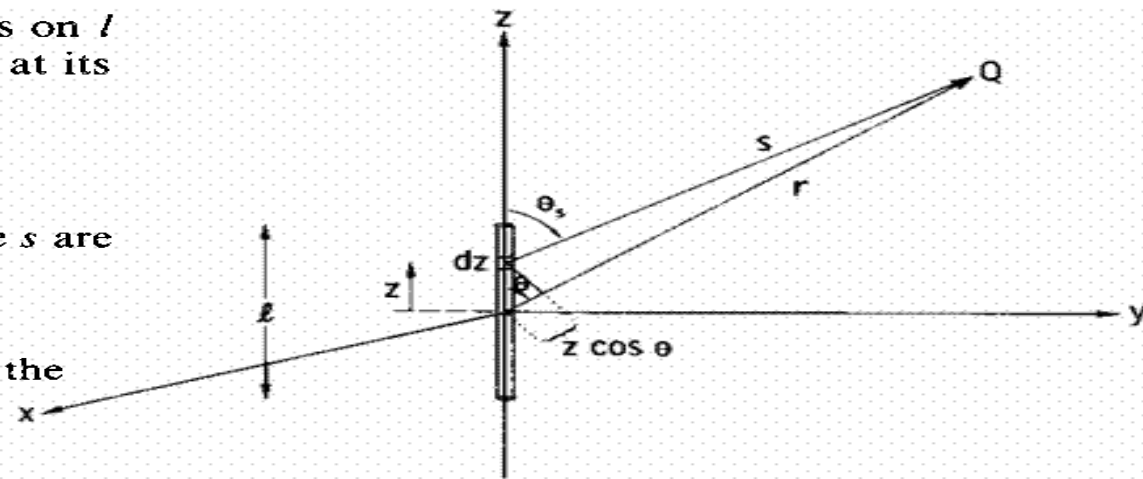
$$E_\theta = \int_{-L/2}^{L/2} dE_\theta.$$

Fig. 3.9   Geometry for linear, center-fed antenna of length l.

Some simplifying approximations can be made to take advantage the far-field conditions.

$$\theta_s \simeq \theta \qquad s = \sqrt{r^2 + z^2 - 2rz\cos\theta} \simeq r - z\cos\theta$$

$$dE_\theta = \frac{jk\eta I}{4\pi r}\, dz\, e^{-jkr} \sin\theta\, e^{jkz\cos\theta}$$

The following expressions for the far-zone fields of the long linear antenna are obtained:

$$E_\theta = \frac{j\eta I_0}{2\pi r}\left[\frac{\cos(\tfrac{1}{2}kl\cos\theta) - \cos(\tfrac{1}{2}kl)}{\sin\theta}\right] e^{-jkr}, \qquad H_\phi = \frac{1}{\eta} E_\theta$$

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Theory – wire antenna example

Once E_θ and E_ϕ are known, the radiation characteristics can be determined.
Defining the directional function f (θ, ϕ) from

$$E_\theta = \frac{e^{-jkr}}{r} f_1(\theta, \phi) \qquad E_\phi = \frac{e^{-jkr}}{r} f_2(\theta, \phi)$$

where $k = 2\pi/\lambda$. The power flow in the far field is then given by

$$S_r = \frac{1}{2\eta r^2} \left(|f_1(\theta, \phi)|^2 + |f_2(\theta, \phi)|^2 \right).$$

Instead of using the power density $S_r(r, \theta, \phi)$ to describe the directional properties of an antenna, it is usually more convenient to use an r-independent function known as the *radiation intensity*, or *radiation pattern*, $F(\theta, \phi)$. This function is given by

$$F(\theta, \phi) = r^2 S_r = \frac{1}{2\eta} \left(|f_1(\theta, \phi)|^2 + |f_2(\theta, \phi)|^2 \right)$$
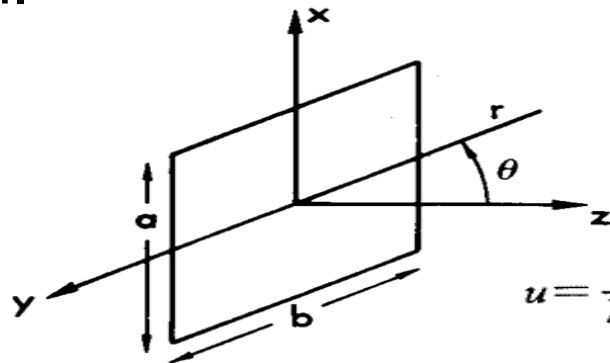
where $F(\theta, \phi)$ is now expressed in watts per unit solid angle (watts per steradian). It is customary to normalize the maximum value of $F(\theta, \phi)$ to unity, in which case the pattern is referred to as the *normalized radiation pattern*, $F_n(\theta, \phi)$. Thus,

$$F_n(\theta, \phi) = \frac{F(\theta, \phi)}{F(\theta, \phi)_{max}} = \frac{S_r(r, \theta, \phi)}{S_r(r, \theta, \phi)_{max}},$$

with the understanding that r is held constant.

Theory – aperture antenna example

The far-field radiation pattern can be found from the Fourier transform of the near-field pattern.

$$u = \frac{x}{\lambda R} = \frac{\sin\theta\cos\phi}{\lambda}$$

$$v = \frac{y}{\lambda R} = \frac{\sin\theta\sin\phi}{\lambda}$$

$$F_n(\theta,\phi) = S_r(\theta,\phi)/S_0$$

$$= \mathrm{sinc}^2(au)\,\mathrm{sinc}^2(bv)$$

$$= \left[\frac{\sin[(\pi a/\lambda)\sin\theta\cos\phi]}{(\pi a/\lambda)\sin\theta\cos\phi} \cdot \frac{\sin[(\pi b/\lambda)\sin\theta\sin\phi]}{(\pi b/\lambda)\sin\theta\sin\phi} \right]^2$$

Where S_r is the radial component of the power density, S_0 is the maximum value of S_r, and F_n is the normalized version of the radiation pattern $F(\theta, \phi)$

$$D_0 = 0.77\left(\frac{4\pi}{\beta_{xz}\,\beta_{yz}} \right)$$
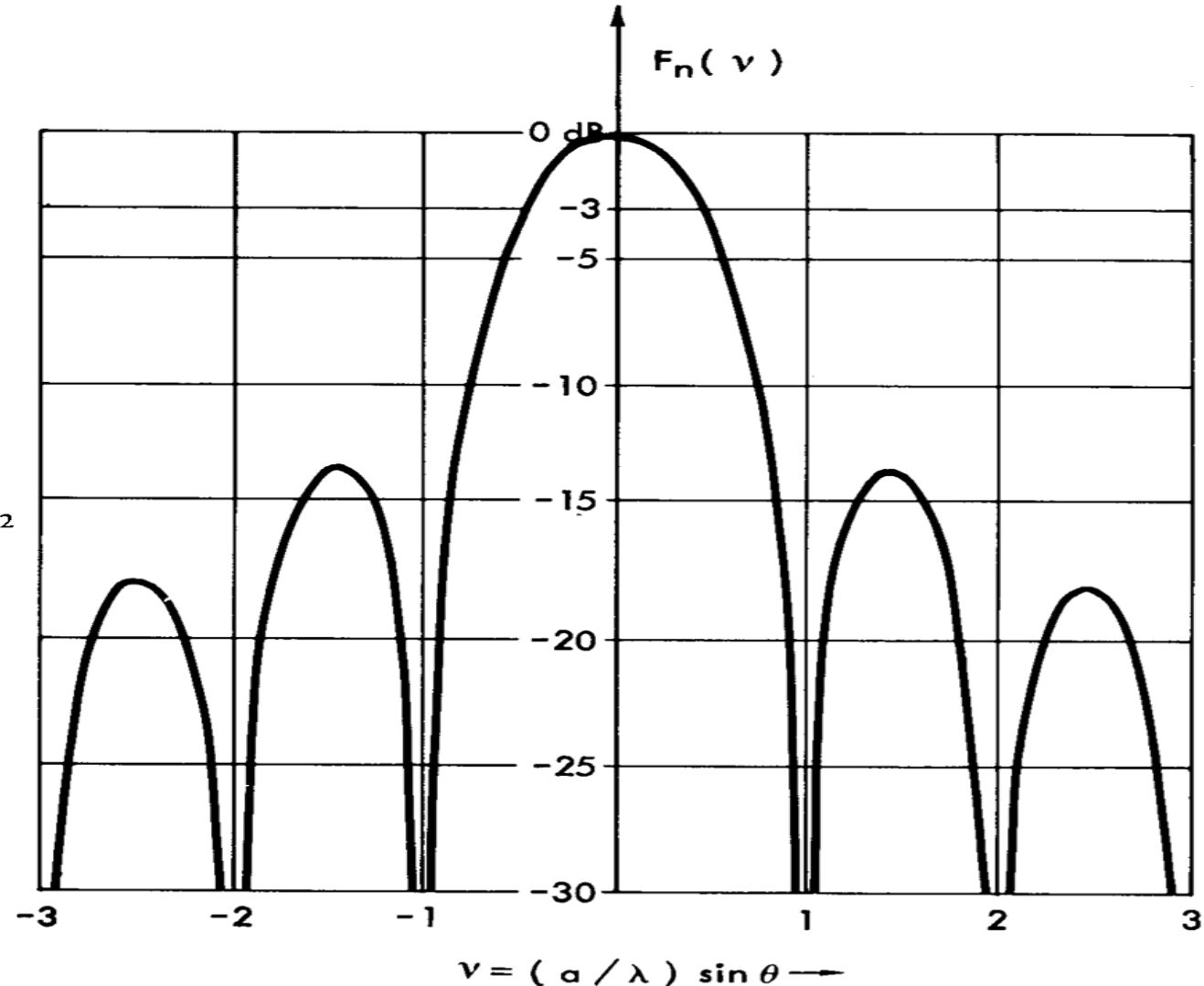
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Fig. 3.15   Normalized radiation pattern of a uniformly illuminated rectangular aperture ($\phi=0$).

Theory

Reciprocity

If an *emf* is applied to the terminals of antenna A and the current measured at the terminals of another antenna B, then an equal current (both in amplitude and phase) will be obtained at the terminals of antenna A if the same *emf* is applied to the terminals of antenna B.

emf: electromotive force, i.e., voltage

Result – the radiation pattern of an antenna is the same regardless of whether it is used to transmit or receive a signal.

ANTENNA PERFORMANCE PARAMETERS
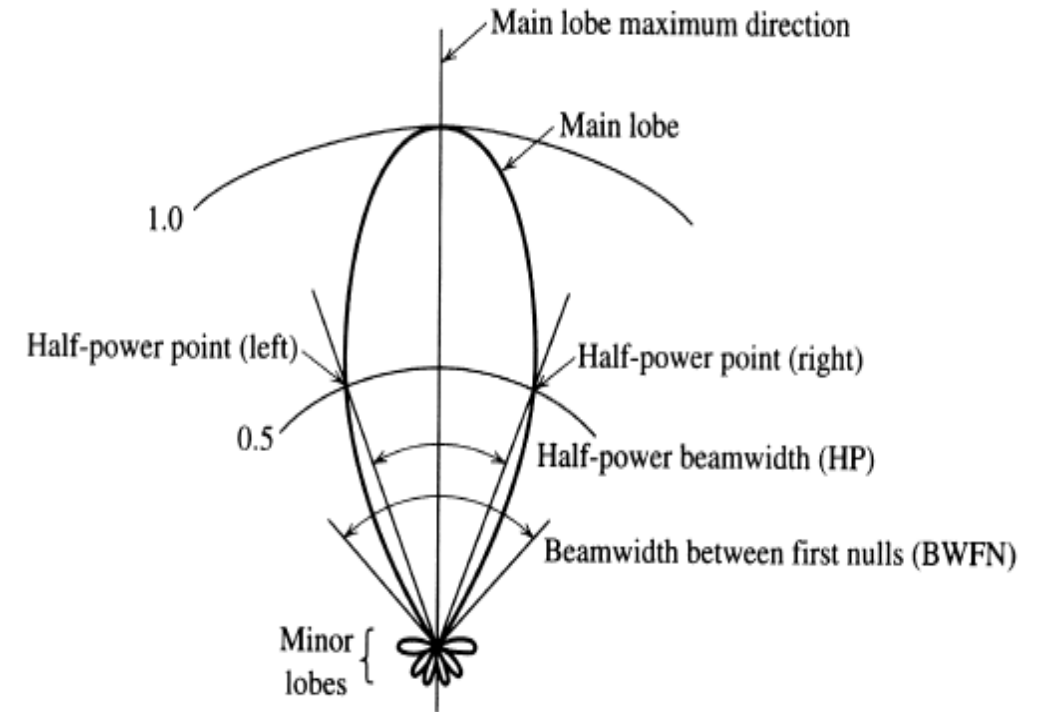
Antenna Performance Parameters

- Common antenna performance parameters include:
  - Gain and Directivity
  - Frequency coverage
  - Bandwidth
  - Beamwidth
  - Polarization
  - Efficiency
  - Field Patterns
  - Impedance
  - Front to Back Ratio and Side loobes

Frequency Coverage and Bandwidth (B)

- The frequency coverage of an antenna is the range of frequencies over which an antenna maintains its parametric performance
  - Antennas are generally rated based upon their stated centre frequency
  - Example:

    9.85-10.15 GHz, f_c = 10.0 GHz

- The bandwidth (B) of an antenna is the frequency range in units of frequency over which the antenna operates
  - Often stated in percentage bandwidth
  - Previous example:

    B = 300 MHz or 3%

Beamwidth (θ_B, Φ_B)

- Beamwidth (θ_B, Φ_B) of an antenna is the angle defined by the points either side of boresight at which the power is reduced by 3-dB, for a given plane.

  - For example if θ_B, represents the beamwidth in the horizontal plane, Φ_B represents the beamwidth in the orthogonal (vertical) plane.

  - The 3-dB (Gücün yarıya düştüğü) beamwidth defines the half-power beam.

Main lobe maximum direction

Main lobe

1.0

Half-power point (left)

Half-power point (right)

0.5

Half-power beamwidth (HP)

Beamwidth between first nulls (BWFN)

Minor lobes

Efficiency (η)

- Total antenna efficiency (η) provides a measure of how much input signal power is output (radiated) by an antenna
  - The two major components are radiation efficiency (η_{rad}) and effective aperture (η_{ap})
  - Losses include spillover, ohmic heating, phase nonconformity, surface roughness and cross-polarization

- $\eta = \eta_{rad} \, \eta_{ap}$

The radiation efficiency (η_{rad}) is a measure of the total power radiated by the antenna (transmitted or received) as compared to the power fed into the antenna

- The aperture efficiency (η_{ap}) is a ratio of the effective aperture area (A_e) and the physical aperture area (A_p). It is a function of the electric field distribution over the aperture.
  - For many antennas this value is close to 0.5

$$\eta_{ap} = A_e / A_p$$

Antennas – Efficiency

Power is fed to an antenna through a T-Line and the antenna appears as a complex impedance

$$Z_{ant} = R_{ant} + jX_{ant}.$$

where the antenna resistance consists of radiation resistance and and a dissipative resistance.

$$R_{ant} = R_{rad} + R_{dis}$$

For the antenna is driven by phasor current

$$I_o = I_s e^{j\alpha}$$

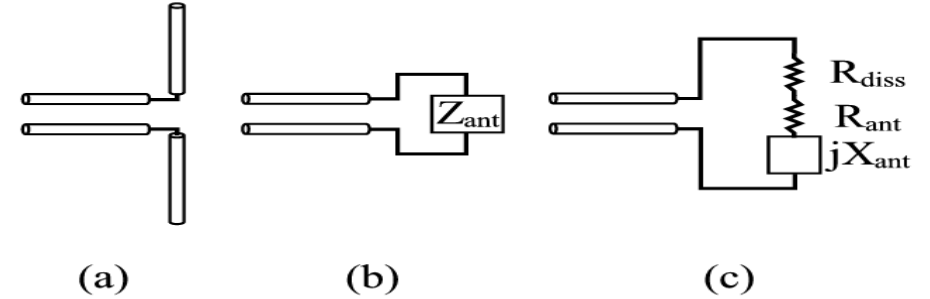The power radiated by the antenna is

$$P_{rad} = \frac{1}{2} I_o^2 R_{rad}$$

The power dissipated by ohmic losses is

$$P_{diss} = \frac{1}{2} I_o^2 R_{diss}$$

An antenna efficiency e can be defined as the ratio of the radiated power to the total power fed to the antenna.

$$e = \frac{P_{rad}}{P_{rad} + P_{diss}} = \frac{R_{rad}}{R_{rad} + R_{diss}}$$

(a) (b) (c)

R_{diss}
R_{ant}
jX_{ant}

Z_{ant}

Antennas – Efficiency

Example

Suppose an antenna has D = 4, R_{rad} = 40 Ω and R_{diss} = 10 Ω. Find antenna efficiency and maximum power gain. (Ans: e = 0.80, G_{max} = 3.2).

Antenna efficiency

$$e = \frac{R_{rad}}{R_{rad} + R_{diss}} = \frac{40}{10 + 40} = 0.8 \text{ (or) } 80\%$$

Maximum power gain

$$G_{max} = eD_{max} = (4)(0.8) = 3.2$$

Maximum power gain in dB

$$G_{max}(dB) = 10 \log_{10}(G_{max}) = 10 \log_{10}(3.2) = 5.05$$

Antenna sitting

- Radio horizon
- Effects of obstacles & structures nearby
- Safety
  - operating procedures
  - Grounding
    - lightning strikes
    - static charges
  - Surge protection
    - lightning searches for a second path to ground

Basic antenna parameters

- Radiation pattern

- Beam area and beam efficiency

- Effective aperture and aperture efficiency

- Directivity and gain

- Radiation resistance

Antennas and Fields

- **Reciprocity Theorem:**
  - An antenna's properties are the same, whether it is used for transmitting or receiving.
- **The Near Field**
  - An electromagnetic field that exists within ~ λ/2 of the antenna. It temporarily stores power and is related to the imaginary term of the input impedance.
- **The Far Field**
  - An electromagnetic field launched by the antenna that extends throughout all space. This field transports power and is related to the radiation resistance of the antenna.

Important Antenna Parameters

- **Feed point impedance (also called input or drive impedance):**
  - Is the impedance measured at the input to the antenna.
  - The real part of this impedance is the sum of the radiation and loss resistances
  - The imaginary part of this impedance represents power temporarily stored by the antenna.
- **Bandwidth**
  - Is the range of frequencies over which one or more antenna parameters stay within a certain range.
  - The most common bandwidth used is the one over which SWR < 2:1

- Antenna Impedance
  - It may be purely resistive, or resistive with a reactive (inductive or capacitive) component.
  - An antenna is said to be *resonant* if it displays no reactive component. That is, its impedance is purely resistive.
  - The resistive portion of the impedance, is made up of a radiation resistance and a loss resistance.
  - The *radiation resistance* is an imaginary resistance. The power "dissipated" in this resistance is the power actually radiated from the antenna.
  - The loss resistance is made up of resistances of the conductors used to make the antenna and other losses in the antenna system. The power dissipated in these resistances is lost, wasted as heat.

SPECIAL CASE OF LOSSLESS TRANSMISSION LINES

TABLE 2.2 Formulas for Transmission Lines

Quantity	General Line	Lossless Line
Propagation constant, $\gamma = \alpha + j\beta$	$\sqrt{(R + j\omega L)(G + j\omega C)}$	$j\omega\sqrt{LC}$
Phase constant, β	$\text{Im}(\gamma)$	$\omega\sqrt{LC} = \dfrac{w}{v} = \dfrac{2\pi}{\lambda}$
Attenuation constant, α	$\text{Re}(\gamma)$	0
Characteristic impedance, Z_0	$\sqrt{\dfrac{R + j\omega L}{G + j\omega C}}$	$\sqrt{\dfrac{L}{C}}$
Input impedance, Z_{in}	$Z_0 \dfrac{Z_L \cosh \gamma l + Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l + Z_L \sinh \gamma l}$	$Z_0 \dfrac{Z_L \cos \beta l + jZ_0 \sin \beta l}{Z_0 \cos \beta l + jZ_L \sin \beta l}$
Impedance of shorted line	$Z_0 \tanh \gamma l$	$jZ_0 \tan \beta l$
Impedance of open line	$Z_0 \coth \gamma l$	$-jZ_0 \cot \beta l$
Impedance of quarter-wave line	$Z_0 \dfrac{Z_L \sinh \alpha l + Z_0 \cosh \alpha l}{Z_0 \sinh \alpha l + Z_L \cosh \alpha l}$	$\dfrac{Z_0^2}{Z_L}$
Impedance of half-wave line	$Z_0 \dfrac{Z_L \cosh \alpha l + Z_0 \sinh \alpha l}{Z_0 \cosh \alpha l + Z_L \sinh \alpha l}$	Z_L
Reflection coefficient, Γ_L	$\dfrac{Z_L - Z_0}{Z_L + Z_0}$	$\dfrac{Z_L - Z_0}{Z_L + Z_0}$
Voltage standing-wave ratio (VSWR)	$\dfrac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$	$\dfrac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$

55

the input impedance of a terminated lossy transmission line is
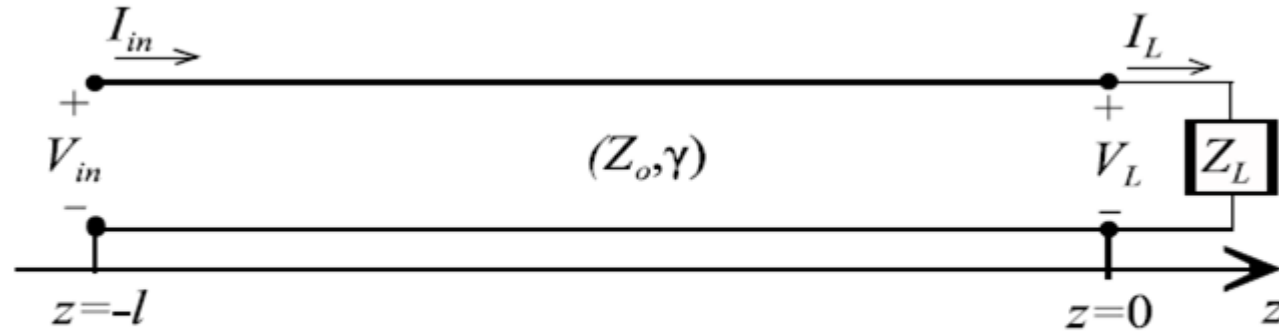
$$Z_{in} = Z_o \frac{Z_L + jZ_o\tan(-j\gamma l)}{Z_o + jZ_L\tan(-j\gamma l)} = Z_o \frac{Z_L + Z_o\tanh(\gamma l)}{Z_o + Z_L\tanh(\gamma l)}$$

The impedance at the input of a transmission line of length l terminated with an impedance Z_L is

$$Z_{in} = Z(-l) = Z_o \frac{Z_L + jZ_o\tan\beta l}{Z_o + jZ_L\tan\beta l}$$

Lossless Transmission Line with Matched Load ($Z_L = Z_o$)
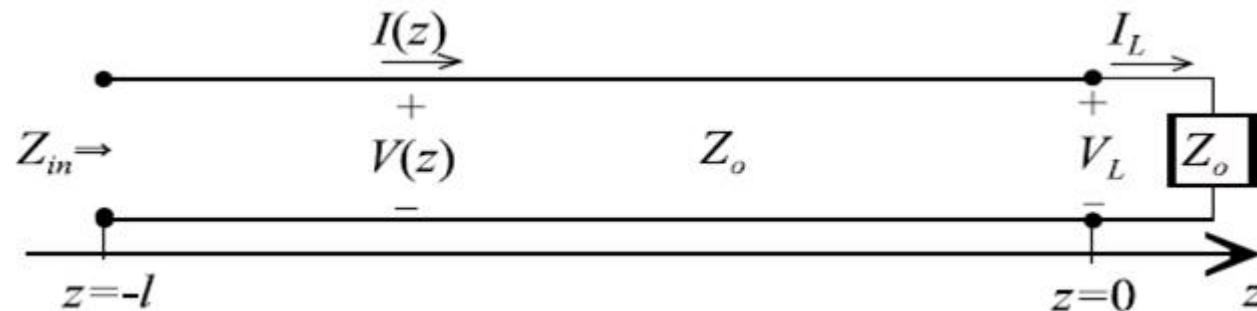
the general impedance at any point along the length of the transmission line may be written as

$$Z(z) = Z_o \frac{e^{-j\beta z} + \Gamma e^{j\beta z}}{e^{-j\beta z} - \Gamma e^{j\beta z}} = Z_o \frac{1 + \Gamma e^{j2\beta z}}{1 - \Gamma e^{j2\beta z}} = Z_o \frac{1 + \Gamma(z)}{1 - \Gamma(z)}$$

The normalized value of the impedance $z_n(z)$ is

$$z_n(z) = \frac{Z(z)}{Z_o} = \frac{1 + \Gamma(z)}{1 - \Gamma(z)} = r(z) + jx(z) \qquad (3)$$

Example $Z_L = 60 + j50$ $Z_o = 50$ $l = \dfrac{4}{10}\lambda$

(b.) $\Gamma = \dfrac{Z_L - Z_o}{Z_L + Z_o} = \dfrac{(60 + j50) - 50}{(60 + j50) + 50} = \dfrac{10 + j50}{110 + j50} = 0.422\angle 54^o$

(a.) $s = \dfrac{1 + |\Gamma|}{1 - |\Gamma|} = \dfrac{1 + 0.422}{1 - 0.422} = 2.46$

(c.) $Y_L = \dfrac{1}{Z_L} = \dfrac{1}{60 + j50} = (9.84 - j8.2)\ \text{mS}$

(d.) $Z_{in} = Z_o\dfrac{Z_L + jZ_o\tan\beta l}{Z_o + jZ_L\tan\beta l}$ $\beta l = \dfrac{2\pi}{\lambda}\dfrac{4}{10}\lambda = \dfrac{4\pi}{5}$

$$= 50\dfrac{(60 + j50) + j50\tan\left(\dfrac{4\pi}{5}\right)}{50 + j(60 + j50)\tan\left(\dfrac{4\pi}{5}\right)} = (24.5 + j20.3)\ \Omega$$

(e.) $\quad |V(z)| = |V_o^+| \, |1 + \Gamma e^{j2\beta z}| \qquad\qquad \Gamma e^{j2\beta z} = |\Gamma| e^{j\theta} e^{j2\beta z}$

$\qquad\quad |V(z)|_{max} = |V_o^+| \, [1 + |\Gamma|] \qquad\quad \theta + 2\beta z_{max} = n\pi \quad \text{(even } n)$

$\qquad\quad |V(z)|_{min} = |V_o^+| \, [1 - |\Gamma|] \qquad\quad \theta + 2\beta z_{min} = n\pi \quad \text{(odd } n)$

$$z_{min} = \frac{n\pi - \theta}{2\beta} = \frac{n\pi - \left(\dfrac{54\pi}{180}\right)}{4\pi}\lambda = \frac{n - 0.3}{4}\lambda \qquad \text{(odd } n)$$

$$n = 1 \quad\Rightarrow\quad z_{min} = \frac{0.7}{4}\lambda = 0.175\lambda$$

$$n = -1 \quad\Rightarrow\quad z_{min} = -\frac{1.3}{4}\lambda = -0.325\lambda \qquad\Rightarrow\qquad l_{min} = 0.325\lambda$$

(f.)

$$z_{max} = \frac{n\pi - \theta}{2\beta} = \frac{n\pi - \left(\dfrac{54\pi}{180}\right)}{4\pi}\lambda = \frac{n - 0.3}{4}\lambda \qquad \text{(even } n)$$

$$n = 0 \quad\Rightarrow\quad z_{max} = -\frac{0.3}{4}\lambda = -0.075\lambda \qquad\Rightarrow\qquad l_{max} = 0.075\lambda$$

$$n = 2 \quad\Rightarrow\quad z_{max} = \frac{1.7}{4}\lambda = 0.425\lambda$$

Antenna temperature

- Power received from antenna as from a black body or the radiation resitance at temperature Ta
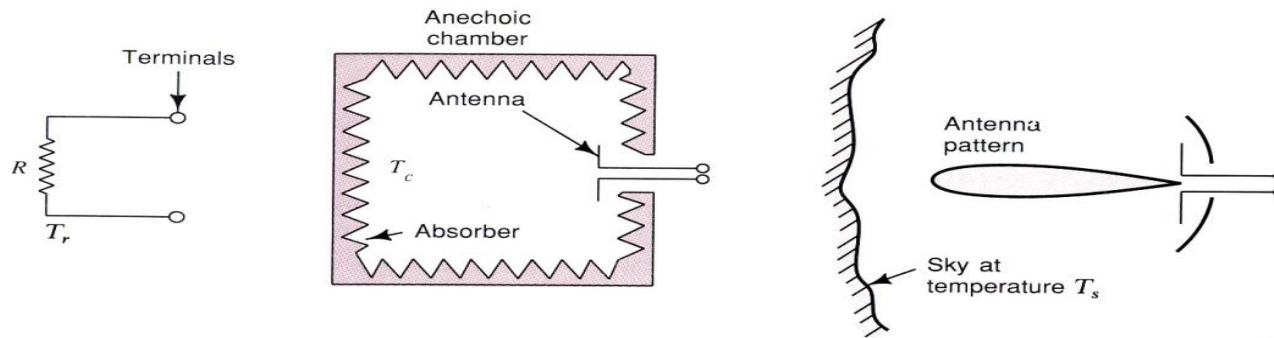
POLARIZATION

Polarization

- A receiving antenna will capture the most energy of a signal when it shares the same polarization with that received signal.
- With a direct or ground wave, this polarization will be the same as the transmitting antenna.
- With a skywave signal, that polarization will be random.
- P gücünün iki bileşini: E ve H; ikisinden ayrı ayrı işaret gönderilir. (Vertical, Horizontal)

Polarization

- The polarization of an antenna defines the orientation of the E and H waves transmitted or received by the antenna
  - Linear polarization includes vertical, horizontal or slant (any angle)
  - Typical non-linear includes right- and left-hand circular (also elliptical)

- The polarization of an antenna in a specific direction is defined to be the polarization of the wave produced by the antenna at a great distance at this direction

Plane-polarized light
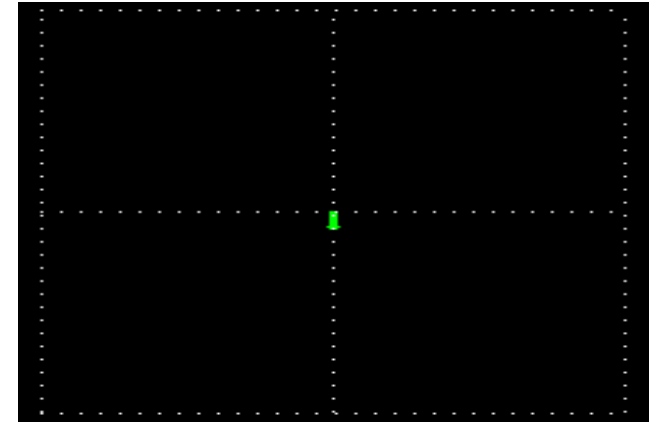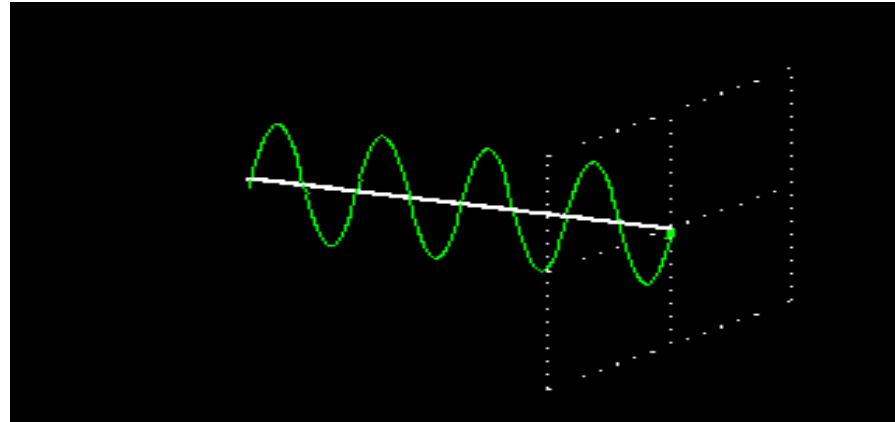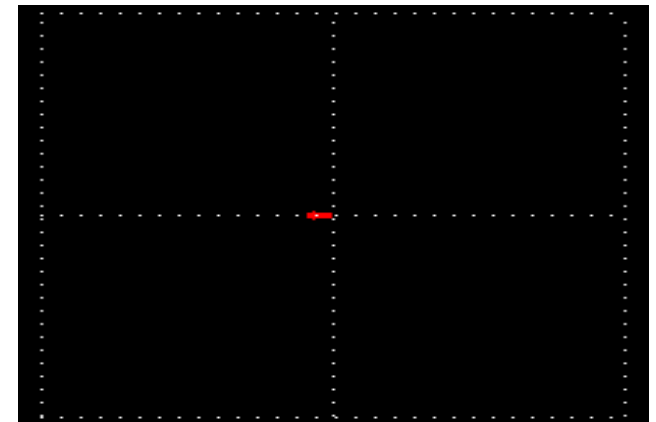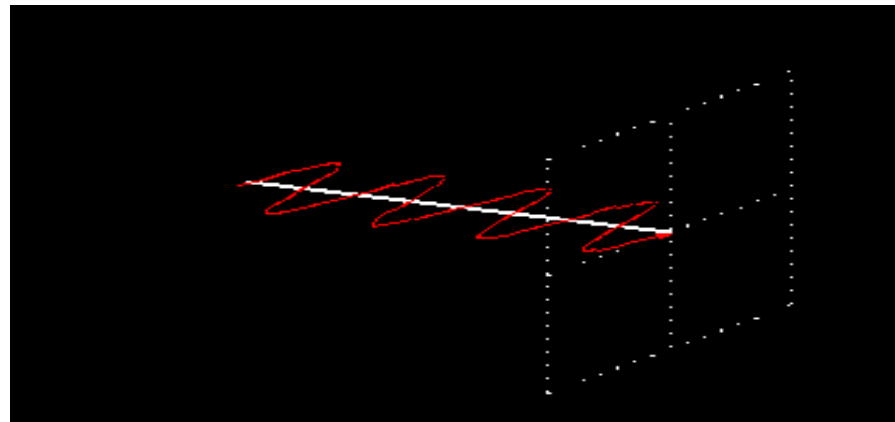
Circularly polarized light

Right circular
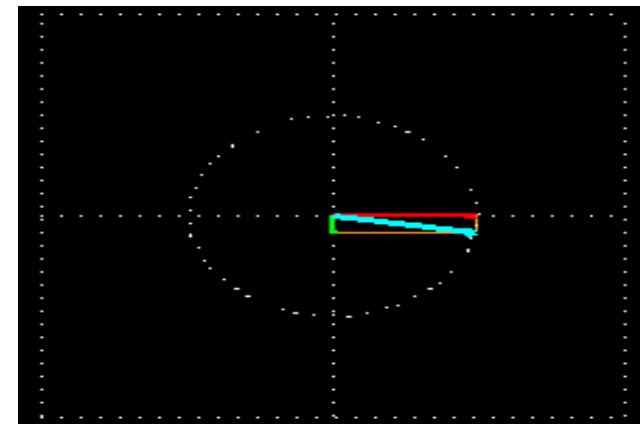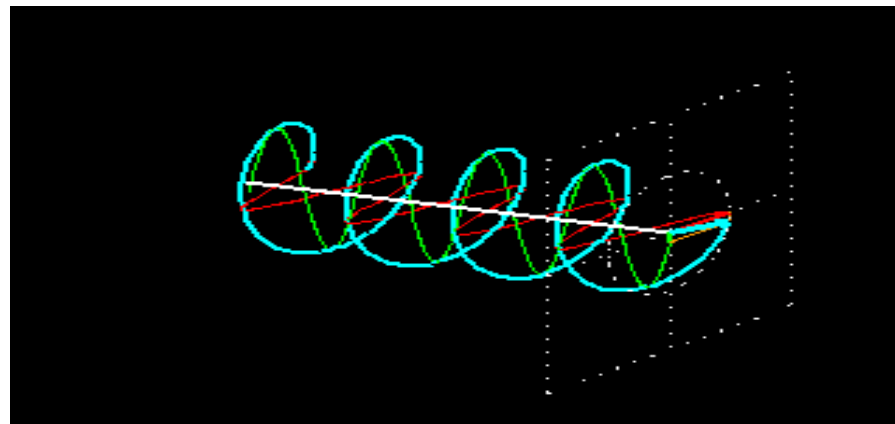

Left circular
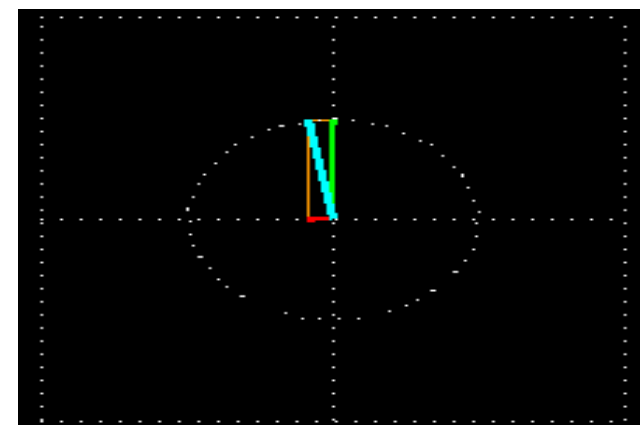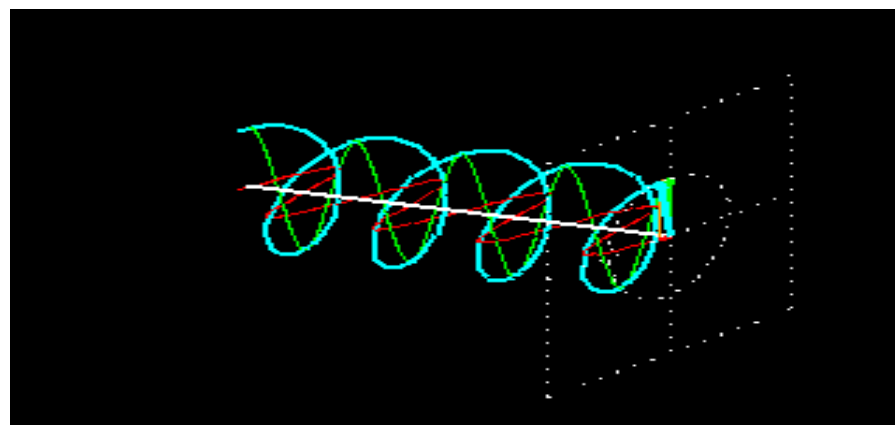
Important Antenna Parameters

- **Directivity or Gain:**
  - **Is the ratio of the power radiated by an antenna in its direction of maximum radiation to the power radiated by a reference antenna in the same direction.**
  - **Is measured in dBi (dB referenced to an isotropic antenna) or dBd (dB referenced to a half wavelength dipole)**

- Antenna Directivity (Gain)
  - Is the ability to direct or focus radiated energy in a specific direction or directions.
  - The measure of the intensity of the directivity is referred to as the gain of the antenna.
  - This gain works for the antenna in receiving signals as well.

Directivity

Directivity

- Ability to focus energy in a specific direction (azimuth and elevation)
  - Power Density of beam not uniform
  - Beamwidth measured at 3 dB down point in az/elev
  - Search Radar - larger beamwidth for detection and tracking
  - Fire Control Radar – smaller beamwidth for accurate targeting solution
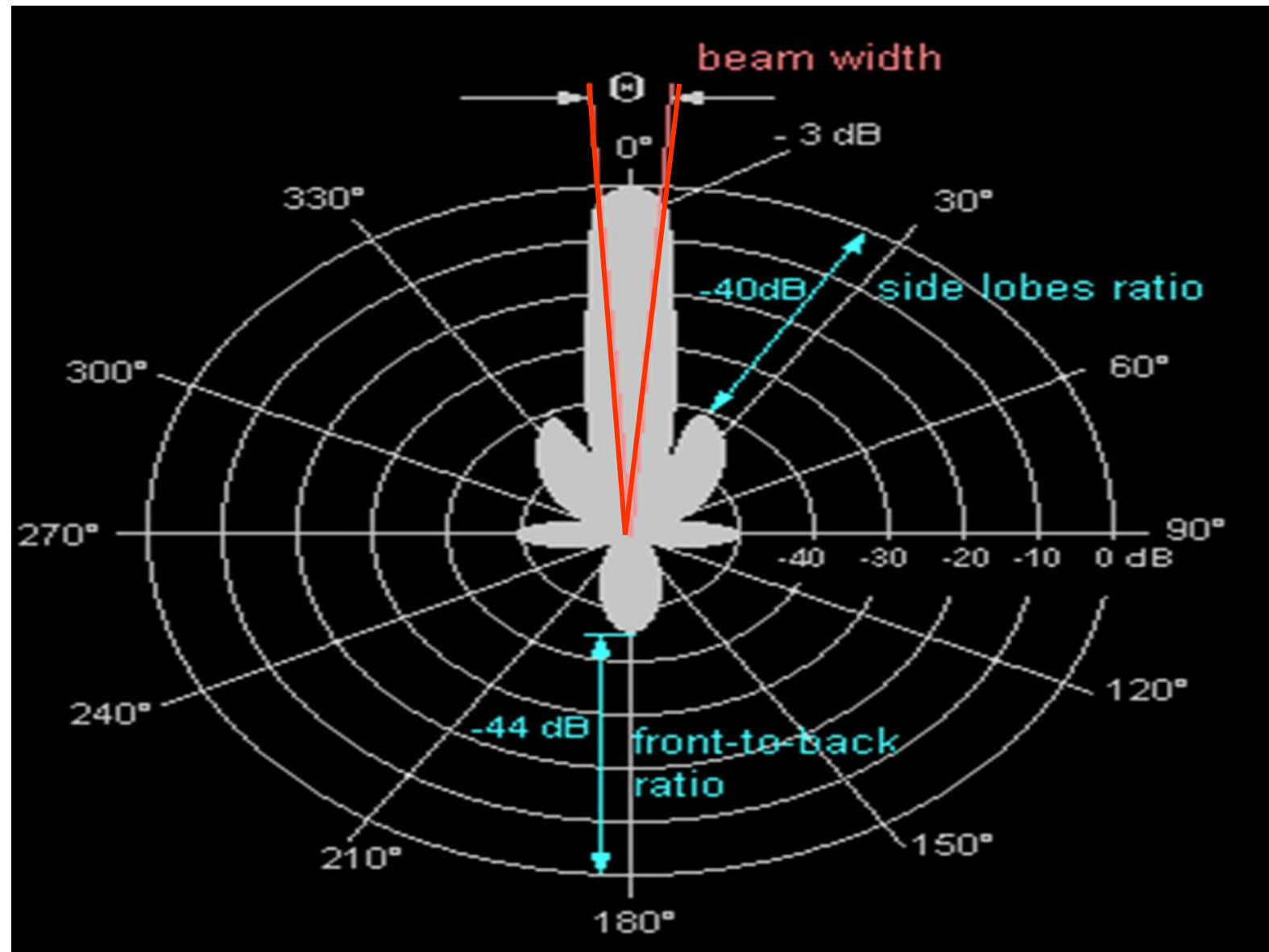
Antennas – Directivity

Directivity:

The *directive gain,,* of an antenna is the ratio of the normalized power in a particular direction to the average normalized power, or

$$D(\theta,\phi) = \frac{P_n(\theta,\phi)}{P_n(\theta,\phi)_{avg}}$$

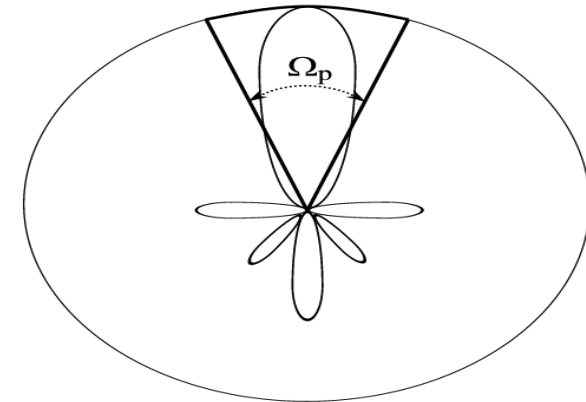Where the normalized power's average value taken over the entire spherical solid angle is

$$P_n(\theta,\phi)_{avg} = \frac{\iint P_n(\theta,\phi)\,d\Omega}{\iint d\Omega} = \frac{\Omega_p}{4\pi}$$

The *directivity, Dmax,* is the maximum directive gain,

$$D_{max} = D(\theta,\phi)_{max} = \frac{P_n(\theta,\phi)_{max}}{P_n(\theta,\phi)_{avg}}$$

$$\boxed{D_{max} = \frac{4\pi}{\Omega_p}}$$ Using $P_n(\theta,\phi)_{max} = 1$

Antennas – Directivity

In free space, suppose a wave propagating radially away from an antenna at the origin has

$$\mathbf{H}_s = \frac{I_s}{r} \sin\theta \; \mathbf{a}_\phi$$

where the driving current phasor

$$I_s = I_o e^{j\alpha}$$

Find (1) $\mathbf{E}_s$

$$\mathbf{E}_s = -\eta_o \mathbf{a}_r \times \mathbf{H}_s = -\eta_o \mathbf{a}_r \times \frac{I_s}{r} \sin\theta \; \mathbf{a}_\phi = -\eta_o \frac{I_s}{r} \sin\theta \left(\mathbf{a}_r \times \mathbf{a}_\phi \right) = \frac{\eta_o I_s}{r} \sin\theta \; \mathbf{a}_\theta$$

Find (2) $\mathbf{P}(r,\theta,\phi)$

$$\mathbf{P}(r,\theta,\phi) = \frac{1}{2}\mathrm{Re}\left[\mathbf{E}_s \times \mathbf{H}_s^* \right] = \frac{1}{2}\mathrm{Re}\left[\left(\frac{\eta_o I_s}{r} \sin\theta \; \mathbf{a}_\theta \right) \times \left(\frac{I_s}{r} \sin\theta \; \mathbf{a}_\phi \right)^* \right]$$

$$= \frac{1}{2}\mathrm{Re}\left[\left(\frac{\eta_o I_o e^{j\alpha}}{r} \sin\theta \; \mathbf{a}_\theta \right) \times \left(\frac{I_o e^{j\alpha}}{r} \sin\theta \; \mathbf{a}_\phi \right)^* \right] = \frac{1}{2}\mathrm{Re}\left[\left(\frac{\eta_o I_o e^{j\alpha}}{r} \sin\theta \; \mathbf{a}_\theta \right) \times \left(\frac{I_o e^{-j\alpha}}{r} \sin\theta \; \mathbf{a}_\phi \right) \right]$$

$$= \frac{1}{2}\mathrm{Re}\left[\eta_o \frac{I_o^2}{r^2} \sin^2\theta \left(\mathbf{a}_\theta \times \mathbf{a}_\phi \right) \right] = \frac{1}{2} \eta_o \frac{I_o^2}{r^2} \sin^2\theta \; \mathbf{a}_r \qquad \text{Magnitude:} \qquad P(r,\theta,\phi) = \frac{1}{2} \eta_o \frac{I_o^2}{r^2} \sin^2\theta$$

Antennas – Directivity

Find (3) P_{rad}

$$P_{rad} = \oiint \mathbf{P}(r,\theta,\phi) \cdot d\mathbf{S} = \int \int P(r,\theta,\phi) r^2 \sin\theta \, d\theta \, d\phi$$

$$P_{rad} = \int \int \left(\frac{1}{2}\eta_o \frac{I_o^2}{r^2}\sin^2\theta \right) r^2 \sin\theta \, d\theta \, d\phi$$

$$P_{rad} = \left(\frac{1}{2}\eta_o \frac{I_o^2}{r^2} \right) \int_0^{2\pi}\int_0^\pi \sin^3\theta \, d\theta \, d\phi$$

$$P_{rad} = \left(\frac{1}{2}\eta_o \frac{I_o^2}{r^2} \right) \left(\int_0^\pi \sin^3\theta \, d\theta \right) \left(\int_0^{2\pi} d\phi \right)$$

$$P_{rad} = = \left(\frac{1}{2}\eta_o \frac{I_o^2}{r^2} \right) \left(\frac{4}{3} \right) (2\pi) = \frac{4}{3}\pi\eta_o I_o^2$$

We make use of the formula

$$\int \sin^3\theta \, d\theta = -\cos\theta + \frac{\cos^3\theta}{3}$$

$$\int_0^\pi \sin^3\theta \, d\theta = \left[-\cos\theta + \frac{\cos^3\theta}{3} \right]_0^\pi$$

$$= \left[\left(-\cos\pi + \frac{\cos^3\pi}{3} \right) - \left(-\cos 0 + \frac{\cos^3 0}{3} \right) \right]$$

$$= \left[\left(1 - \frac{1}{3} \right) - \left(-1 + \frac{1}{3} \right) \right] = 2 - \frac{2}{3} = \frac{4}{3}$$

Find (4) $P_n(r,\theta,\phi)$ Normalized Power Pattern

$$P(r,\theta,\phi) = \frac{1}{2}\eta_o \frac{I_o^2}{r^2}\sin^2\theta$$

$$P_{max} = \frac{1}{2}\eta_o \frac{I_o^2}{r^2}$$

$$P_n(\theta,\phi) = \frac{P(r,\theta,\phi)}{P_{max}}$$

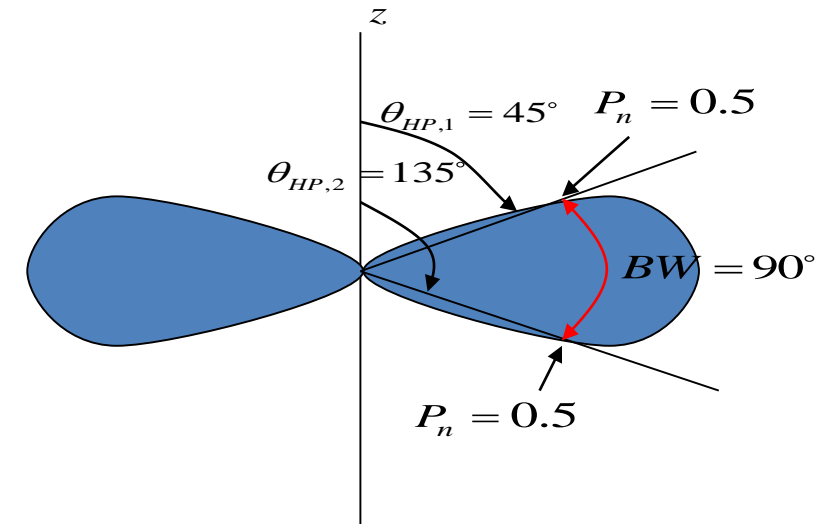$$P_n(\theta,\phi) = \sin^2\theta$$

Antennas – Directivity

Find (5) Beam Width

$$P_n(\theta, \phi) = \sin^2 \theta \implies \frac{1}{2} = \sin^2 \theta_{HP} \qquad \sin \theta_{HP} = \pm \frac{1}{\sqrt{2}}$$

$$\sin \theta_{HP} = + \frac{1}{\sqrt{2}}$$

$$\theta_{HP,1} = 45° \text{ and } \theta_{HP,2} = 135°$$

$$Beamwidth \, (BW) = 135° - 45° = 90°$$

$\theta_{HP,1} = 45°$ $P_n = 0.5$

$\theta_{HP,2} = 135°$

$BW = 90°$

$P_n = 0.5$

(6) Pattern Solid Angle Ω_p (Integrate over the entire sphere!)

$$\boxed{\Omega_p = \iint P_n(\theta, \phi) \, d\Omega}$$

$$\Omega_P = \iint \sin^2 \theta \sin \theta \, d\theta \, d\phi = \int_0^{2\pi} \int_0^{\pi} \sin^3 \theta \, d\theta \, d\phi = \left(\int_0^{\pi} \sin^3 \theta \, d\theta \right) \left(\int_0^{2\pi} d\phi \right) = \left(\frac{4}{3} \right)(2\pi) = \frac{8\pi}{3}$$

(7) directivity D_{max}

$$D_{max} = \frac{4\pi}{\Omega_P} = \frac{4\pi}{\dfrac{8\pi}{3}} = \frac{2}{3} = 1.5$$

Antennas – Directivity

(8) Half-power Pattern Solid Angle $\Omega_{p,HP}$ (Integrate over the beamwidth!)

$$\boxed{\Omega_{p,HP} = \int\int P_n(\theta,\phi)\, d\Omega}$$
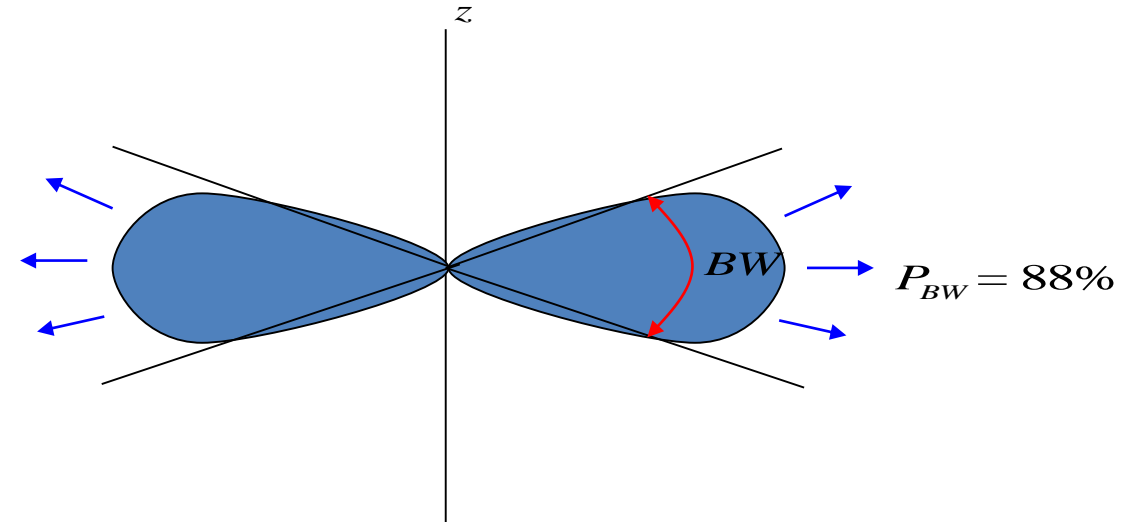
$$\Omega_{P,HP} = \int\int \sin^2\theta \sin\theta\, d\theta\, d\phi = \int_0^{2\pi}\int_{45°}^{135°} \sin^3\theta\, d\theta\, d\phi = \left(\int_{45°}^{135°}\sin^3\theta\, d\theta\right)\left(\int_0^{2\pi} d\phi\right) = \left(\frac{5}{3\sqrt{2}}\right)(2\pi) = \frac{5\pi\sqrt{2}}{3}$$

$$\int_{45°}^{135°}\sin^3\theta\, d\theta = \left[-\cos\theta + \frac{\cos^3\theta}{3}\right]_{45°}^{135°} = \left[\left(-\cos(135°)+\frac{\cos^3(135°)}{3}\right) - \left(-\cos(45°)+\frac{\cos^3(45°)}{3}\right)\right]$$

$$= \left[\left(\frac{1}{\sqrt{2}}-\frac{1}{6\sqrt{2}}\right) - \left(-\frac{1}{\sqrt{2}}+\frac{1}{6\sqrt{2}}\right)\right] = \frac{2}{\sqrt{2}} - \frac{2}{6\sqrt{2}} = \frac{10}{6\sqrt{2}} = \frac{5}{3\sqrt{2}}$$

Power radiated through the beam width

$$P_{BW} = \frac{\Omega_{P,HP}}{\Omega_P} = \frac{\dfrac{5\pi\sqrt{2}}{3}}{\dfrac{8\pi}{3}} = \frac{5\sqrt{2}}{8} \cong 0.88 \text{ (or) } 88\%$$
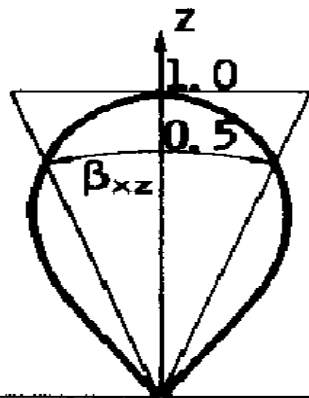

BW

$P_{BW} = 88\%$

Beamwidth and beam solid angle

The *beam* or *pattern solid angle*, Ω_p [steradians or sr] is defined as
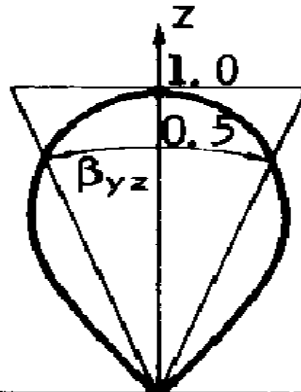
$$\Omega_p = \iint\limits_{4\pi} F_n(\theta, \phi)\, d\Omega$$

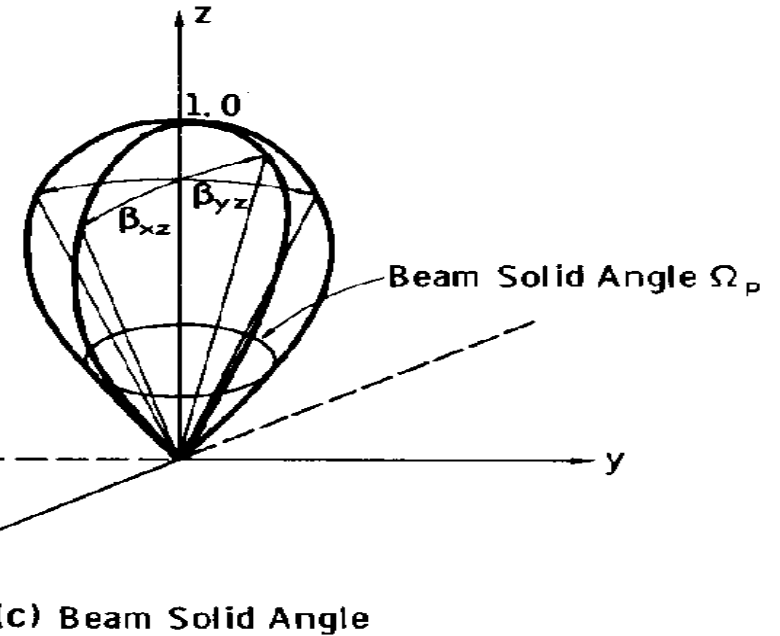where $d\Omega$ is the elemental solid angle given by

$$d\Omega = \sin\theta\, d\theta\, d\phi$$

(a) Half-Power Beamwidth in x–z Plane

(b) Half-Power Beamwidth in y–z Plane

(c) Beam Solid Angle

Fig. 3.5 The solid angle of a unidirectional radiation pattern is approximately equal to the product of the half-power beamwidths in the two principal planes, i.e., $\Omega_p \cong \beta_{xz}\beta_{yz}$.

Directivity, gain, effective area

Directivity – the ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions.

$$D(\theta,\phi) = \frac{F_n(\theta,\phi)}{\dfrac{1}{4\pi}\displaystyle\iint_{4\pi} F_n(\theta,\phi)\,d\Omega} \qquad \text{[unitless]}$$

Maximum directivity, D_o, found in the direction (θ, ϕ) where $F_n = 1$

$$D_0 = \frac{4\pi}{\displaystyle\iint_{4\pi} F_n(\theta,\phi)\,d\Omega} = \frac{4\pi}{\Omega_p} \qquad \text{and} \qquad \Omega_p \simeq \beta_{xz}\beta_{yz} \qquad \text{or} \qquad D_0 = \frac{4\pi}{\Omega_p} \simeq \frac{4\pi}{\beta_{xz}\beta_{yz}}$$

Given D_o, D can be found

$$D(\theta,\phi) = D_0\, F_n(\theta,\phi)$$

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Antenna Gain

Directivity, gain, effective area

Effective area – the functional equivalent area from which an antenna directed toward the source of the received signal gathers or absorbs the energy of an incident electromagnetic wave

It can be shown that the maximum directivity D_o of an antenna is related to an *effective area* (or *effective aperture*) A_{eff}, by

$$D_0 = \frac{4\pi}{\lambda^2} A_{eff} = \frac{4\pi}{\lambda^2} \eta_a A_p$$

where A_p is the physical aperture of the antenna and $\eta_a = A_{eff}/A_p$ is the *aperture efficiency* ($0 \leq \eta_a \leq 1$)
Consequently

$$A_{eff} = \frac{\lambda^2}{\Omega_p} \cong \frac{\lambda^2}{\beta_{xz}\,\beta_{yz}} \qquad [\text{m}^2]$$

For a rectangular aperture with dimensions l_x and l_y in the x- and y-axes, and an aperture efficiency $\eta_a = 1$, we get

$$\beta_{xz} \cong \lambda/l_x \qquad [\text{rad}] \qquad\qquad \beta_{yz} \cong \lambda/l_y \qquad [\text{rad}]$$

Directivity, gain, effective area

Therefore the maximum gain and the effective area can be used interchangeably by assuming a value for the radiation efficiency (e.g., $\eta_l = 1$)

$$G_O = \frac{4\pi}{\lambda^2}\, \eta_l\, A_{eff}$$

$$G_O \cong A_{eff}\, \frac{4\pi}{\lambda^2} = \frac{4\pi}{\beta_{xz}\,\beta_{yz}}$$

$$A_{eff} \cong G_O\, \frac{\lambda^2}{4\pi}$$

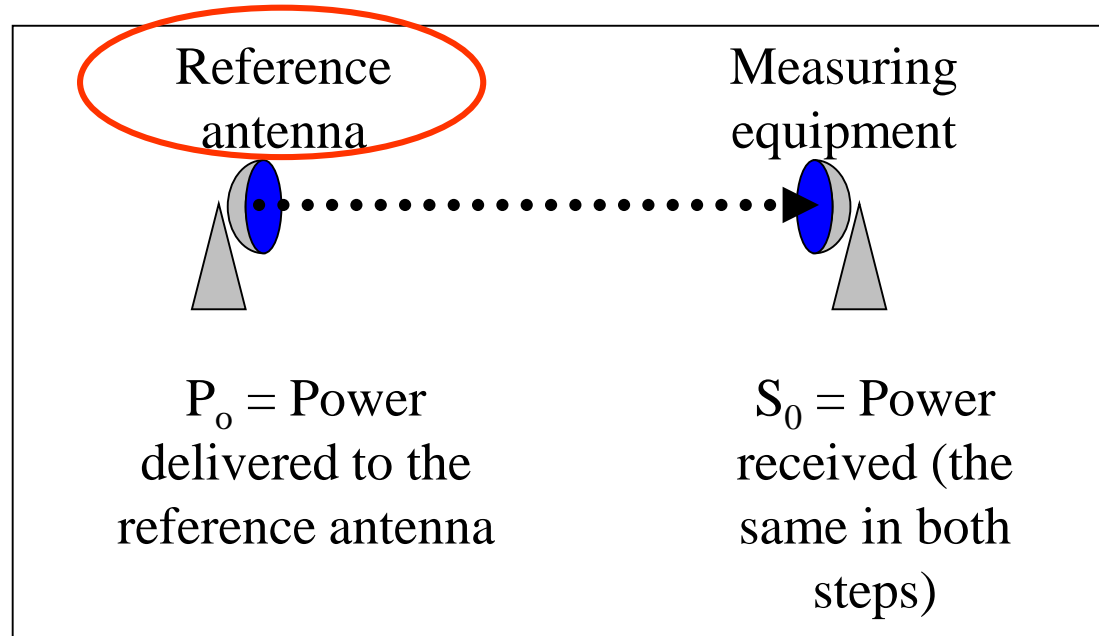**Example:** For a 30-cm x 10-cm aperture, f = 10 GHz (λ = 3 cm)

$\beta_{xz} \cong$ 0.1 radian or 5.7°, $\beta_{yz} \cong$ 0.3 radian or 17.2°

$G_0 \cong$ 419 or 26 dBi

(dBi: dB relative to an isotropic radiator)

Type of Antenna	Effective Area A_e (m^2)	Power Gain (relative to isotropic)
Isotropic	$\lambda^2/4\pi$	1
Infinitesimal dipole or loop	$1.5\ \lambda^2/4\pi$	1.5
Half-wave dipole	$1.64\ \lambda^2/4\pi$	1.64
Horn, mouth area A	$0.81\ A$	$10\ A/\lambda^2$
Parabolic, face area A	$0.56\ A$	$7\ A/\lambda^2$
Turnstile (two crossed, perpendicular dipoles)	$1.15\ \lambda^2/4\pi$	1.15

Antenna gain measurement

Step 1: reference

Reference antenna — Measuring equipment

P_o = Power delivered to the reference antenna

S_0 = Power received (the same in both steps)

Step 2: substitution

Actual antenna — Measuring equipment

P = Power delivered to the actual antenna

S = Power received (the same in both steps)
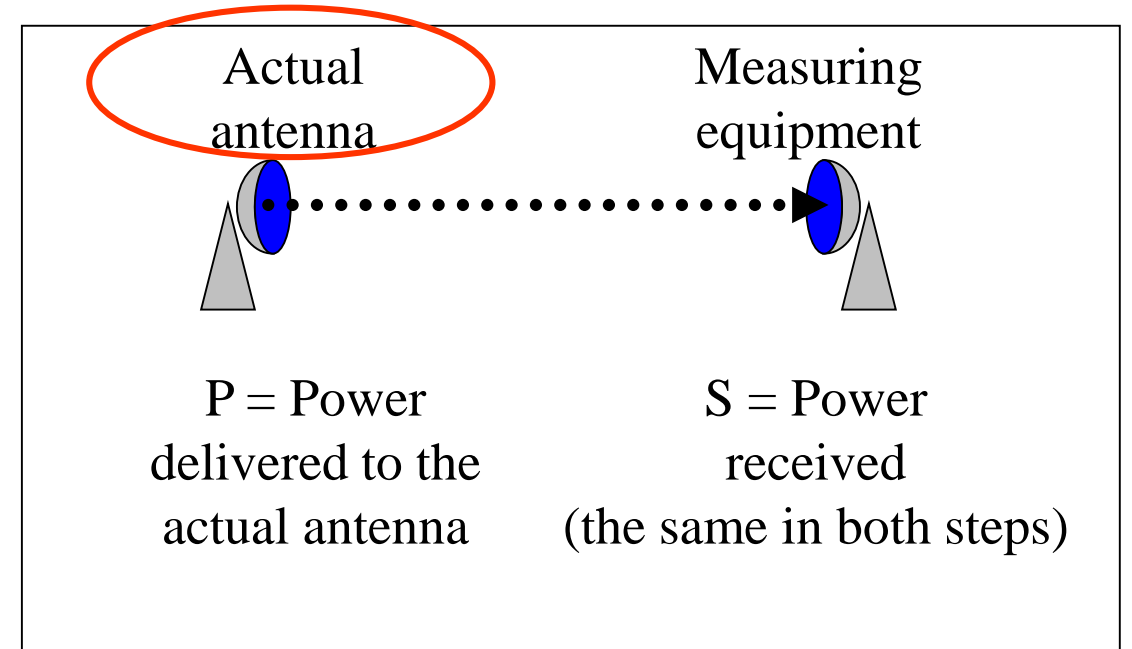
$$\text{Antenna Gain} = (P/P_o)_{S=S0}$$

Kazancı Bilinmeyen Bir Anten Kazancının Hesaplanması

- Pr1 ölçüldü, Pr1=Pt+Gt1+Gr-Lt-Lr-FSL

- Pr2 ölçüldü, Pr2=Pt+Gt2+Gr-Lt-Lr-FSL

- Gt2 hesaplanır, Gt2=Gt1 + Pr2-Pr1

dBi

Isotropic Antenna

The **isotropic antenna** radiates equally in all directions

Radiation pattern is spherical

This is a theoretical antenna that cannot be built.

Elevation pattern

Azimuth pattern

Dipole Antenna

λ/2-dipole

Feed

$\lambda/2$

A dipole can be of any length, but the antenna patterns shown are only for the λ/2-dipole.
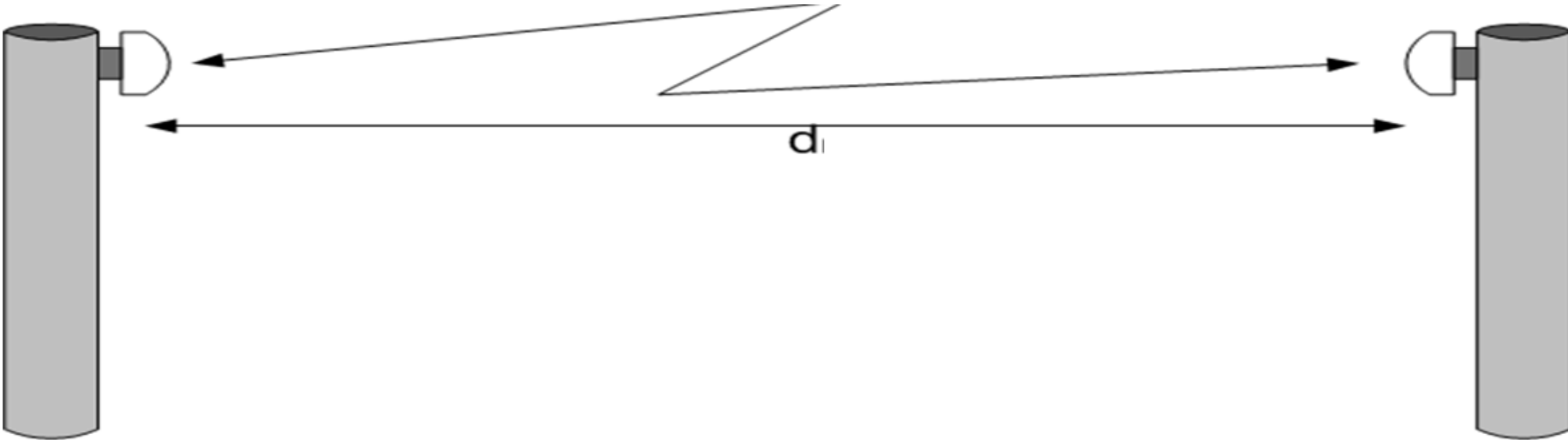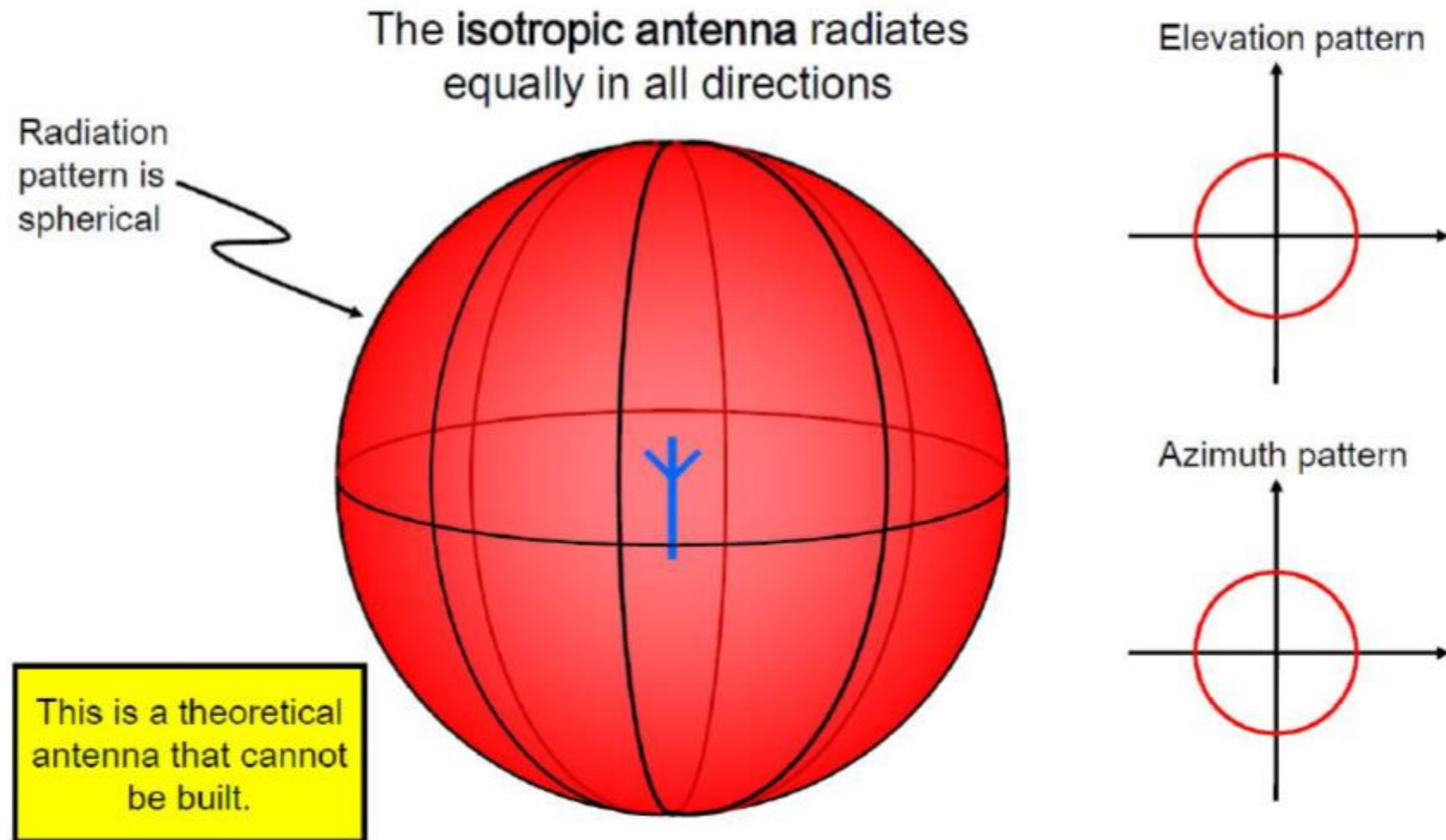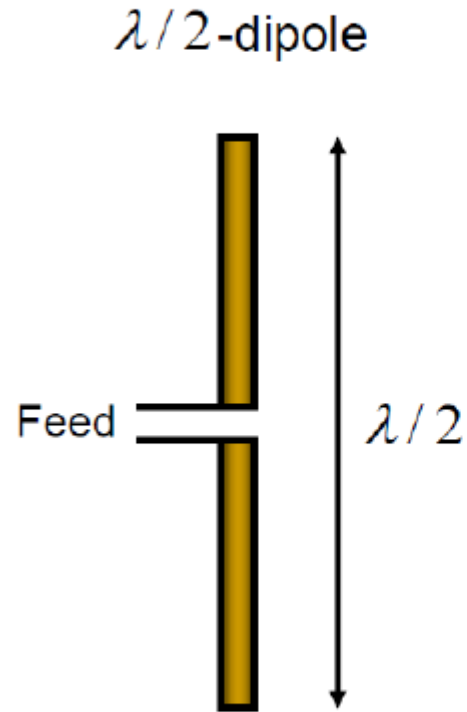
This antenna does not radiate straight up or down. Therefore, more energy is available in other directions.

THIS IS THE PRINCIPLE BEHIND WHAT IS CALLED *ANTENNA GAIN*.

Elevation pattern
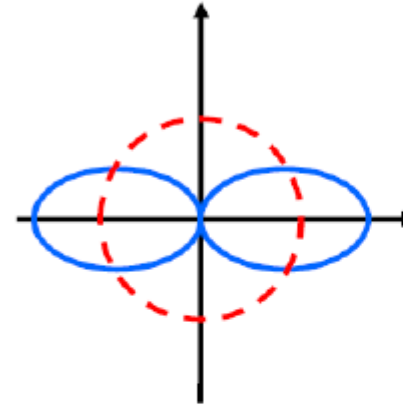
Azimuth pattern
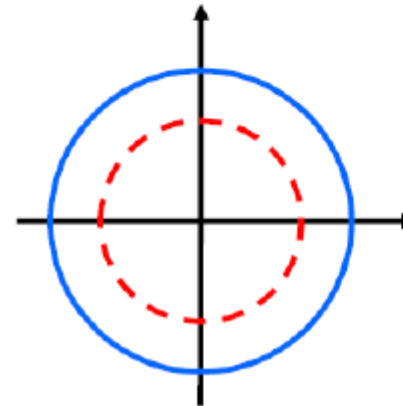
- - - Antenna pattern of isotropic antenna.

Gain_Tot[dB]

| 0.0 |
| -2.0 |
| -4.0 |
| -6.0 |
| -8.0 |
| -10.0 |
| -12.0 |
| -14.0 |
| -16.0 |
| -18.0 |
| -20.0 |

www.antenna-theory.com
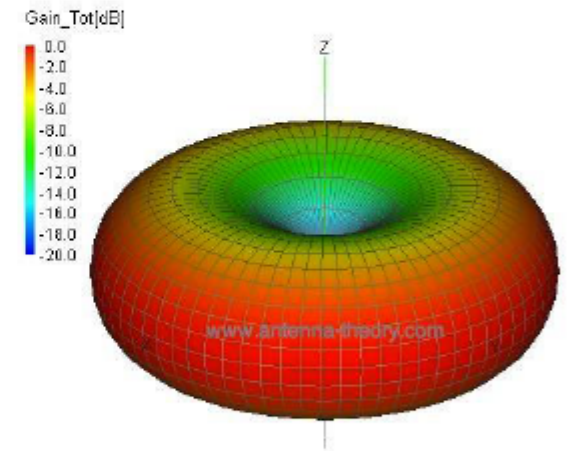
Dipole Antenna

Antenna gain is a relative measure.

We will use the isotropic antenna as the reference.

Radiation pattern

Isotropic and dipole,
with equal input
power!

Isotropic, with increased
input power.

The amount of increase
in input power to the
isotropic antenna, to
obtain the same maximum
radiation is called the
antenna gain!

Antenna gain of the λ/2 dipole is **2.15 dB**.
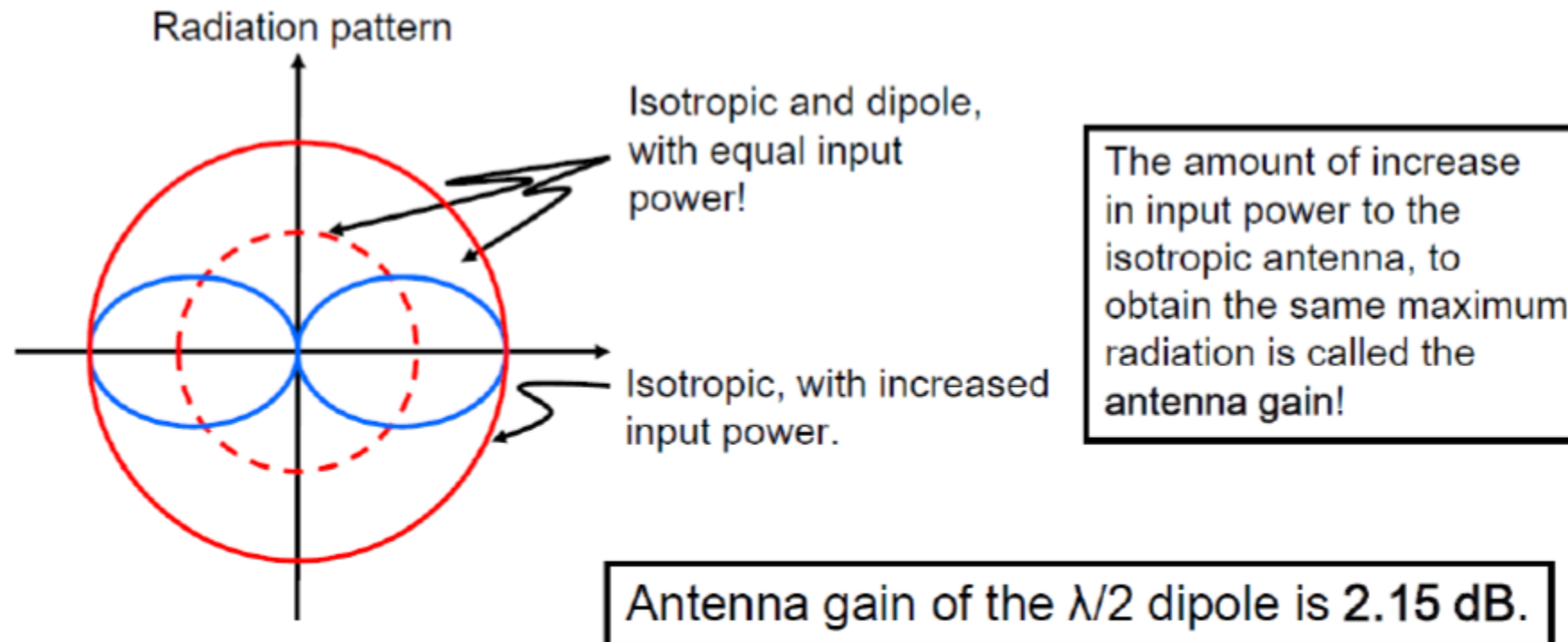
dBi

Sometimes the notation **dBi** is used for antenna gain (instead of dB).

The "**i**" indicates that it is the gain relative to the
isotropic antenna (which we will use in this course).

Another measure of antenna gain frequently encountered
is **dBd**, which is relative to the λ/2 dipole.

$$G\big|_{dBi} = G\big|_{dBd} + 2.15$$

Be careful! Sometimes it is not clear if the antenna gain is given in dBi or dBd.

dBi versus dBd

•**dBi** indicates gain vs. isotropic antenna
   •Isotropic antenna radiates equally well in all directions,
    spherical pattern, Gain=1, Gain(dBi))=0 dBi

•**dBd** indicates gain vs. reference half-wavelength dipole
   •Dipole has a doughnut shaped pattern with a gain of 2.15 dBi
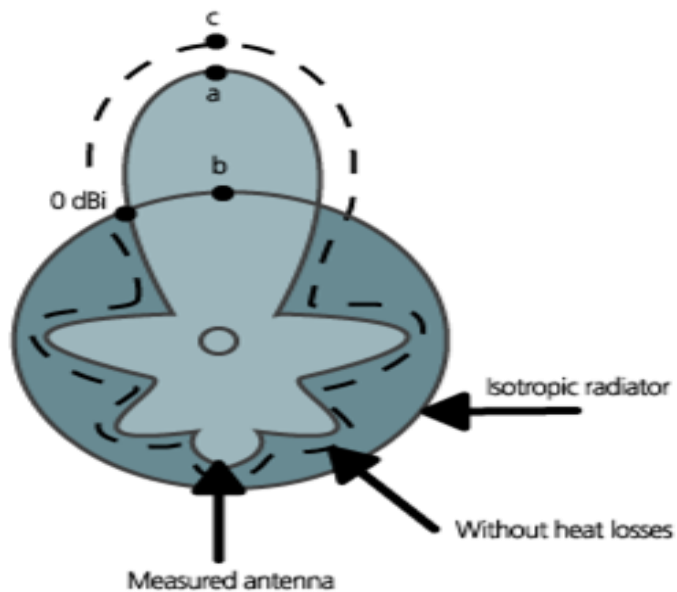
$$dBi = dBd + 2.15\,dB$$

Directivity and gain

Directivity

$$D = \frac{P(\theta,\phi)_{max}}{P(\theta,\phi)_{average}}$$

From pattern

$$D = \frac{4\pi}{\displaystyle\iint_{4\pi} P_n(\theta,\phi)\,d\Omega} = \frac{4\pi}{\Omega_A}$$

From aperture

$$D = 4\pi \frac{A_e}{\lambda^2}$$

Isotropic antenna:

$$\Omega_A = 4\pi \qquad D = 1$$

Gain $\quad G = k_g\, D$

k_g = efficiency factor $(0 < k_g < 1)$

G is lower than D due to ohmic losses only

Antenna Gain

- Relationship between antenna gain and effective area

$$G = \frac{4\pi A_e}{\lambda^2} = \frac{4\pi f^2 A_e}{c^2}$$

  - G = antenna gain
  - A_e = effective area
  - f = carrier frequency
  - c = speed of light (3×10^8 m/s)
  - λ = carrier wavelength

1. The Gain of an antenna with losses is given by:

$$G \simeq \frac{4\pi\eta A}{\lambda^2}$$

Where
- η = Efficiency
- A = Physical aperture area
- λ = wavelength

another is:

$$G = \frac{X\,\eta}{BW_\phi\, BW_\theta}$$

Where $BW_{\theta\,and\,\phi}$ are the elev & az beamwidths in degrees.
For approximating an antenna pattern with:
(1) A rectangle; $X = 41253$, $\eta_{typical} = 0.7$
(2) An ellipsoid; $X = 52525$, $\eta_{typical} = 0.55$

2. Gain of rectangular X-Band Aperture

   $G = 1.4\,LW$ Where: Length (L) and Width (W) are in cm

3. Gain of Circular X-Band Aperture

   $G = d^2\eta$ Where: d = antenna diameter in cm

   η = aperture efficiency

4. Gain of an isotropic antenna radiating in a uniform spherical pattern is one (0 dB).

5. Antenna with a 20 degree beamwidth has a 20 dB gain.

6. 3 dB beamwidth is approximately equal to the angle from the peak of the power to the first null (see figure at right).

Antenna Radiation Pattern

7. Parabolic Antenna Beamwidth:

   $$BW = \frac{70\lambda}{d}$$

   Where: BW = antenna beamwidth; λ = wavelength; d = antenna diameter.
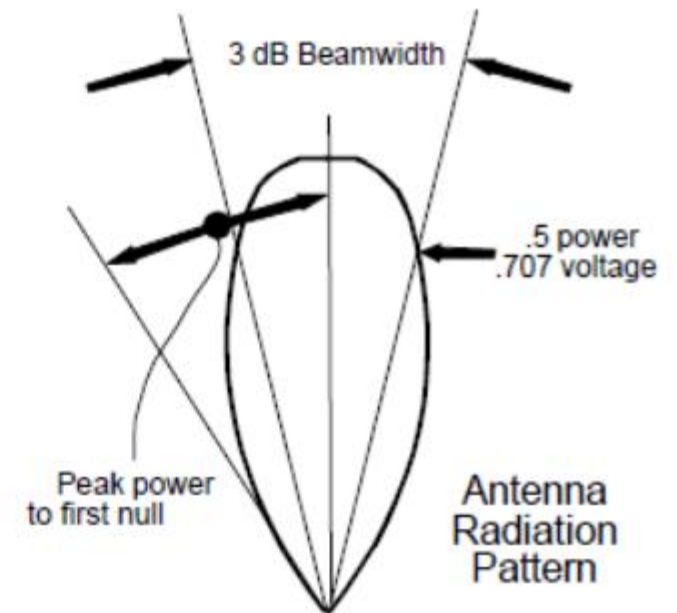
Power Transfer in Free Space

Power Transfer in Free Space

$$P_R = PFD \cdot A_e$$

$$= \left(\frac{G_T P_T}{4\pi r^2} \right) \left(\frac{\lambda^2 G_R}{4\pi} \right)$$

$$= P_T G_T G_R \left(\frac{\lambda}{4\pi r} \right)^2$$

- λ: wavelength [m]
- P_R: power available at the receiving antenna
- P_T: power delivered to the transmitting antenna
- G_R: gain of the transmitting antenna in the direction of the receiving antenna
- G_T: gain of the receiving antenna in the direction of the transmitting antenna
- Matched polarizations

Signal transmission, radar echo

- Transmitting antenna $\qquad A_{et}, P_t, G_t, \lambda$

- Receiving antenna $\qquad A_{er}, P_r, G_r$

$$P_r = \frac{G_t P_t}{4\pi \cdot r^2} \frac{\lambda^2 G_r}{4\pi} = \left(\frac{\lambda}{4\pi \cdot r}\right)^2 G_t G_r P_t$$

S, power density $\qquad$ Effective receiving area

Radar return
$$P_r = \frac{G_t P_t}{4\pi \cdot r^2} \frac{\sigma}{4\pi \cdot r^2} \frac{G_r \lambda^2}{4\pi} = P_t G_t G_r \frac{\lambda^2}{(4\pi)^3 r^4} \sigma$$

S, power density $\qquad$ Reflected power density $\qquad$ Effective receiving area

$$\sigma = \text{radar cross section (area)}$$

Antenna Radiation

Antennas

- You would change a dipole antenna to make it resonant on a higher frequency by making it shorter.
- The electric field of vertical antennas is perpendicular to the Earth.
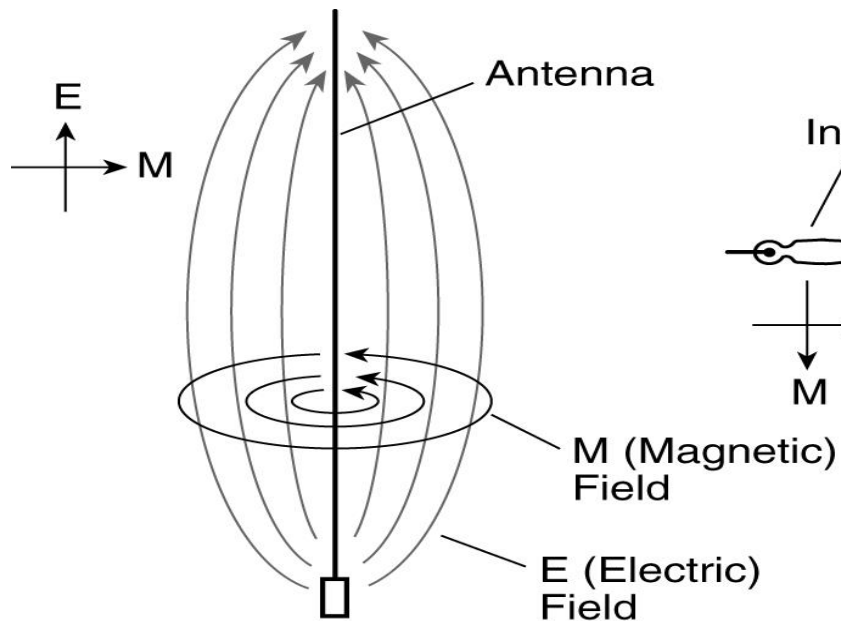
a. Vertically-Polarized Antenna
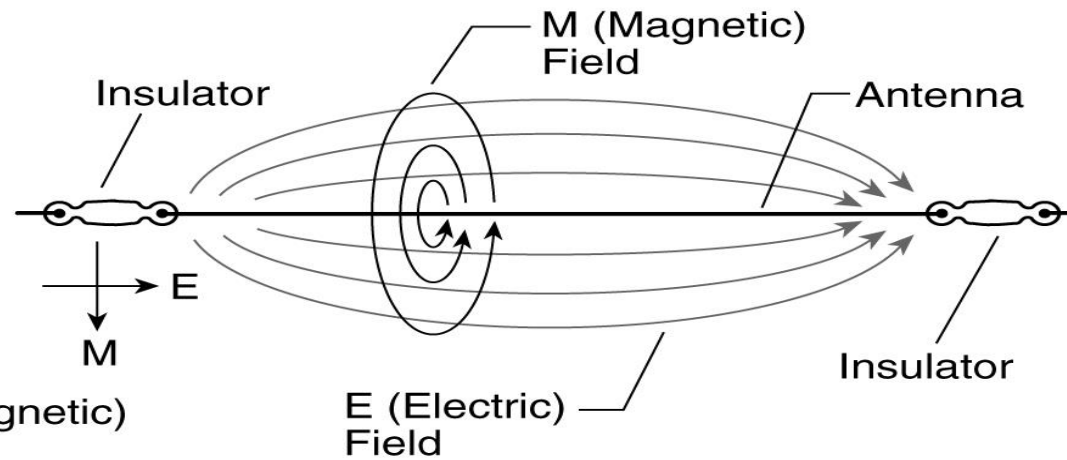
b. Horizontally-Polarized Antenna

Vertical and Horizontal Polarization

H & V Polarized Antennas

Antenna Basics

$$G = \frac{P_{directional}}{P_{isotropic}}$$

Isotropic

0 dB$_i$

Dipole

2.2 dB$_i$

High gain
directional

14 dB$_i$
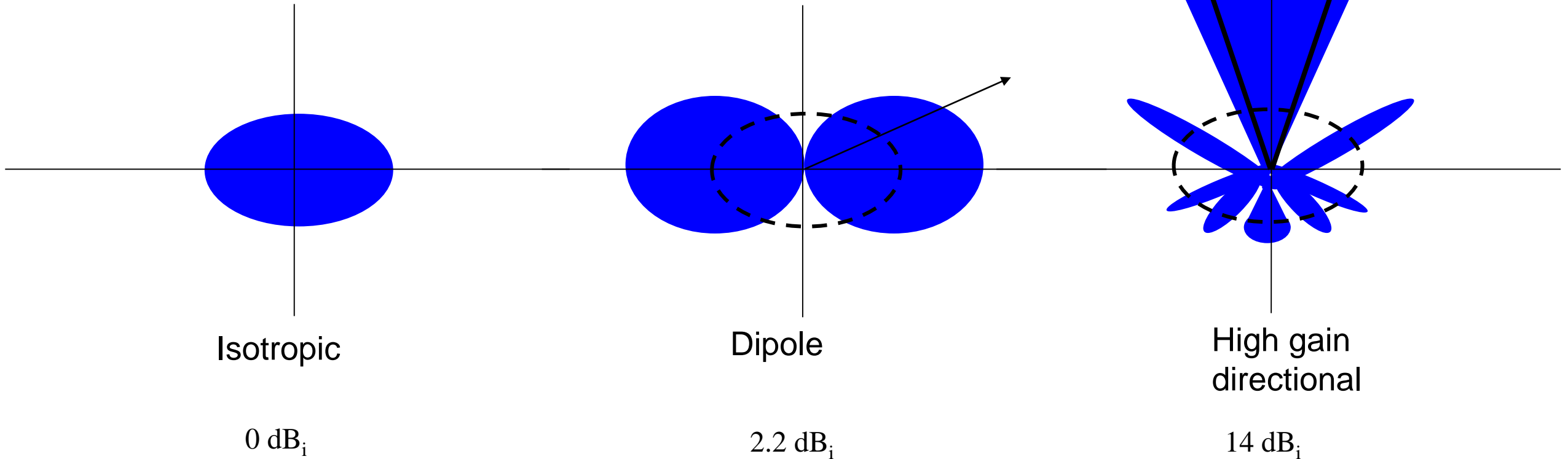
Omnidirectional Antenna
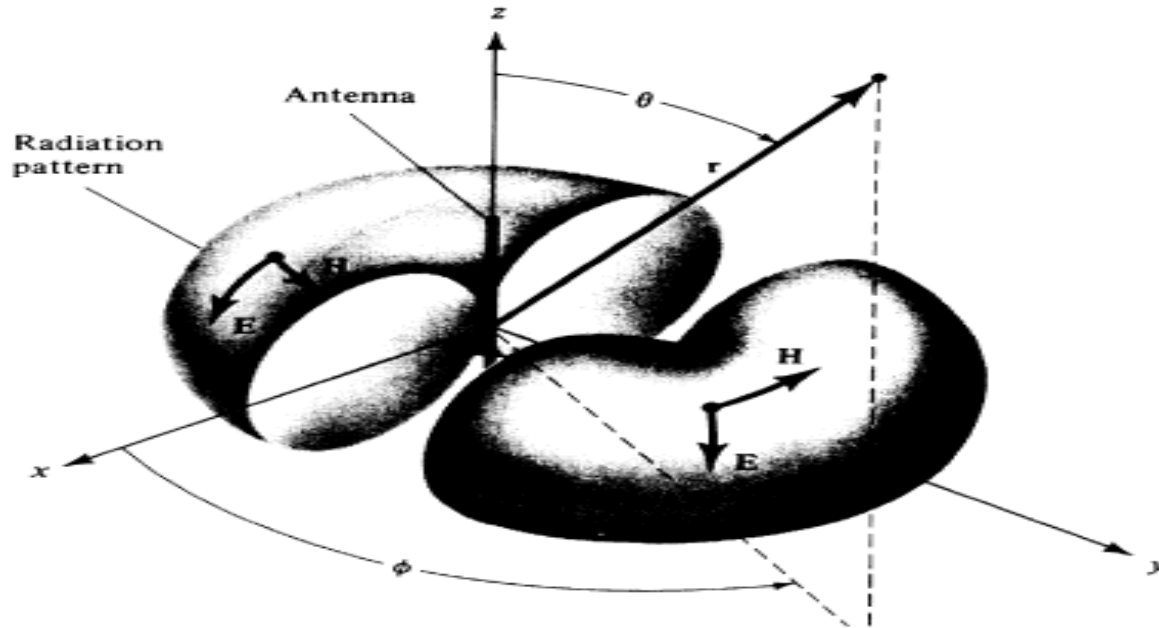
- An antenna, which has a non-directional pattern in a plane
  - It is usually directional in other planes

Radiation & Induction Fields

- There are two *induction fields* or areas where signals collapse and radiate from the antenna. They are known as the *near field* and *far field*. The distance that antenna inductance has on the transmitted signal is directly proportional to antenna height and the dimensions of the wave

$$R > \frac{2D^2}{\lambda}$$

Where: R = *the distance from the antenna*

D = *dimension of the antenna*

λ = *wavelength of the transmitted signal*

Antenna Field Regions

Far–field region
(Fraunhofer region)

Radiating near–field region
(Fresnel region)

Reactive
near–field region

D

R_1

R_2
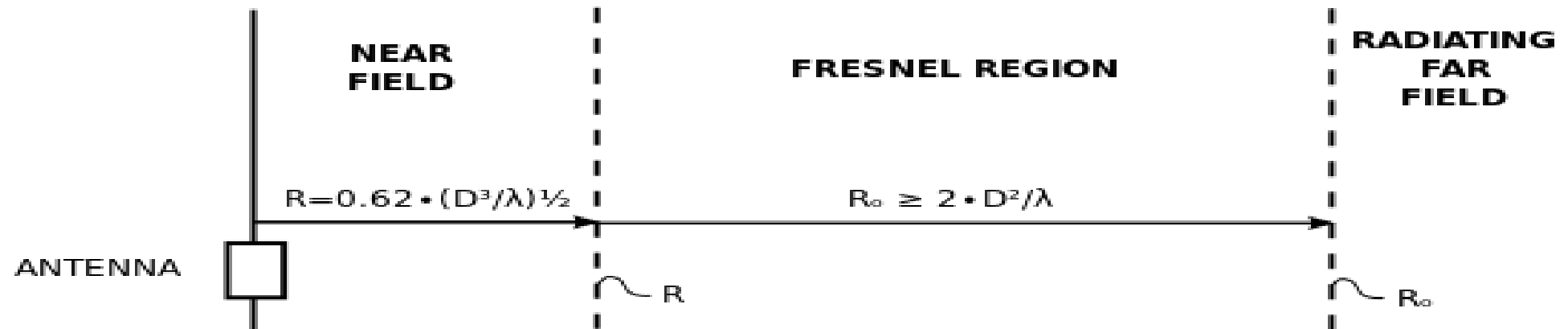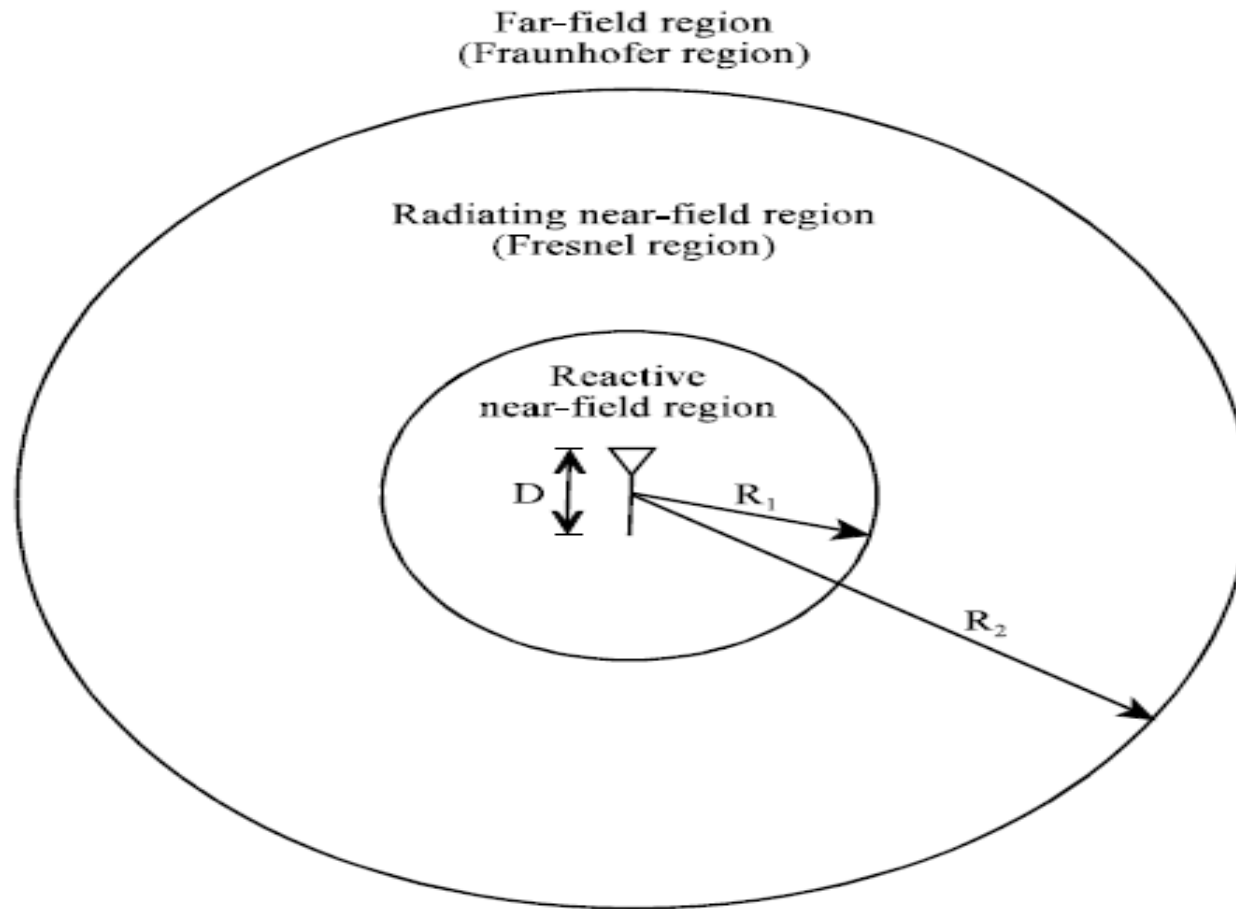
D = maximum antenna dimension

$$R_1 = 0.62\sqrt{\frac{D^3}{\lambda}}$$

$$R_2 = \frac{2D^2}{\lambda}$$

What are the **types** of field zone?

The **fields** around **an antenna** may be divided into two principal regions.

- Near field zone (**Fresnel** zone)
- Far field zone (Fraunhofer zone)

Boundary sphere of antenna region

$$R = \frac{2L^2}{\lambda}$$

To infinity

Far field or Fraunhofer region

Near field or Fresnel region

Antenna region

L

Fresnel-Fraunhofer boundary sphere
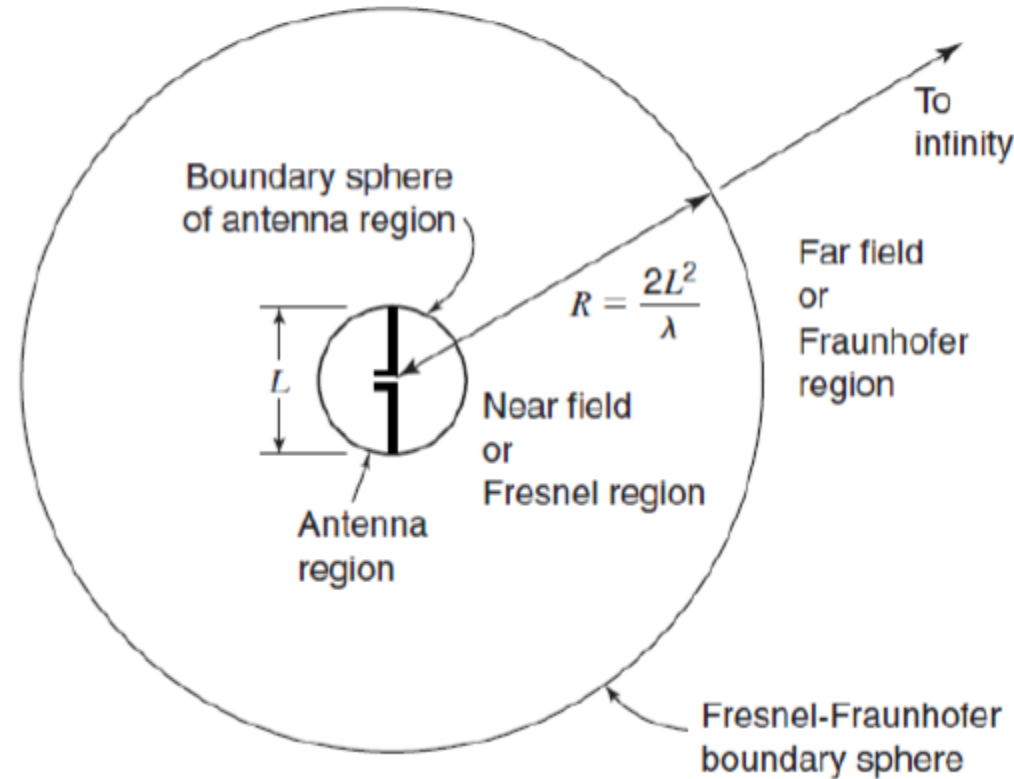
Radiation Pattern

Radiation Pattern

The radiation pattern of an antenna is a graphical representation of the radiation properties of the antenna. Graphically, we surround the antenna by a sphere and evaluate the electric / magnetic fields (far field radiation fields) at a distance equal to the radius of the sphere.
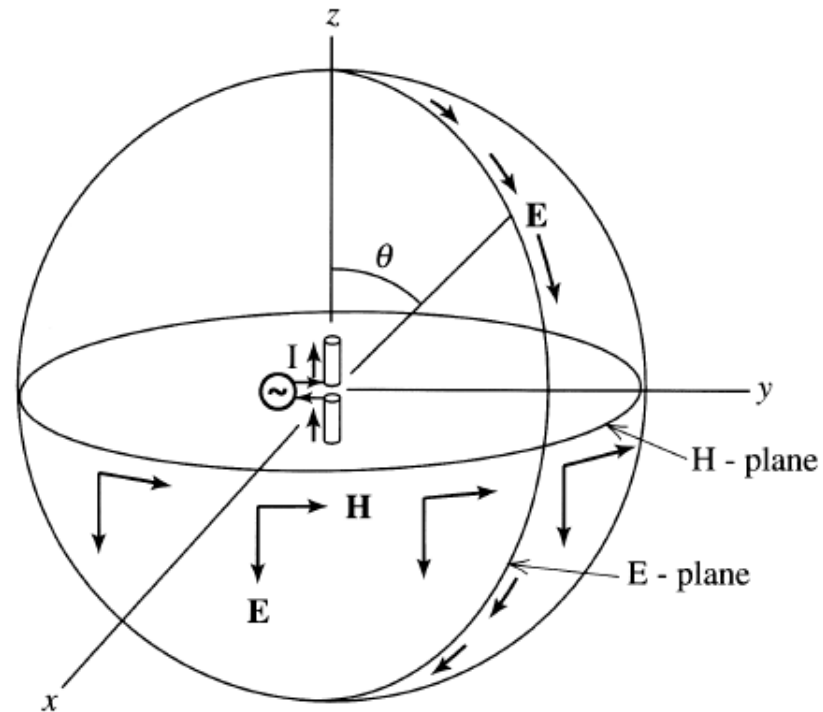
Radiated fields evaluated on an imaginary sphere surrounding a dipole

- Antenna radiation pattern (*antenna pattern*):
  - is defined for large distances from the antenna, where the spatial (angular) distribution of the radiated power does not depend on the distance from the radiation source
  - is independent on the power flow direction: it is the same when the antenna is used to transmit and when it is used to receive radio waves
  - is usually different for different frequencies and different polarizations of radio wave radiated/ received

Power pattern vs. Field pattern

Antenna under test

Auxiliary antenna

Large distance

Power or field-strength meter

Generator

Turntable
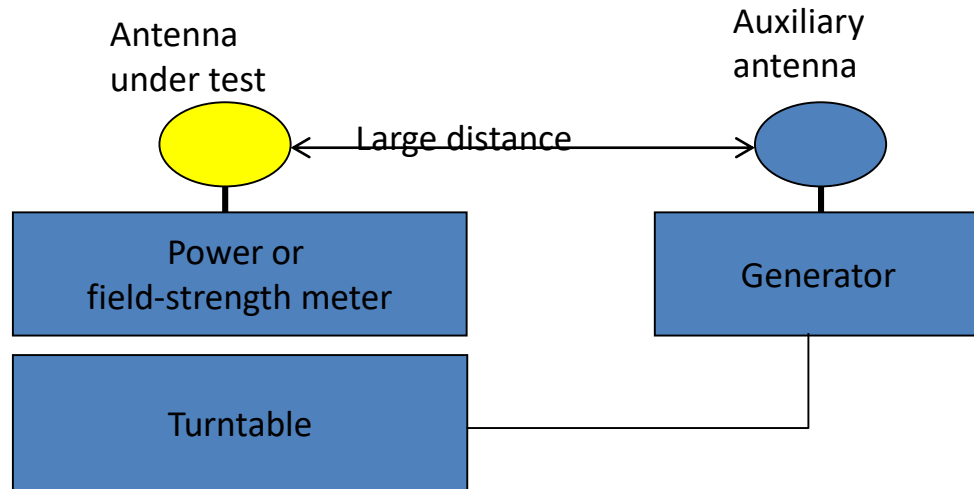
- The *power pattern* is the measured (calculated) and plotted received power: $|P(\theta, \varphi)|$ at a constant (large) distance from the antenna

- The *amplitude field pattern* is the measured (calculated) and plotted electric (magnetic) field intensity, $|E(\theta, \varphi)|$ or $|H(\theta, \varphi)|$ at a constant (large) distance from the antenna

- The power pattern and the field patterns are inter-related:
  $P(\theta, \varphi) = (1/\eta)*|E(\theta, \varphi)|^2 = \eta*|H(\theta, \varphi)|^2$

  P = power

  E = electrical field component vector

  H = magnetic field component vector

  η = 377 ohm (free-space, plane wave impedance)

Antennas – Radiation Patterns

Radiation Pattern:

A directional antenna radiates and receives preferentially in some directic

It is customary, then, to take slices of the pattern and generate two-dimensional plots.

The polar plot can also be in terms of decibels.
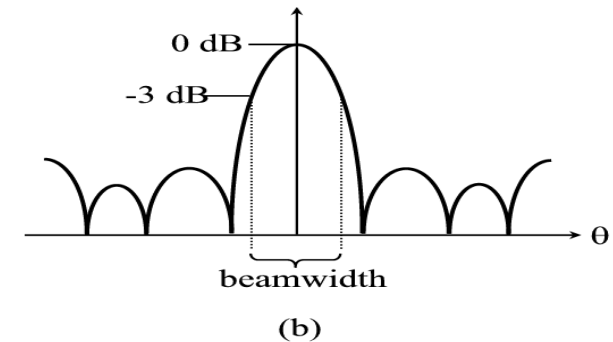
$$E_n(\theta,\phi) = \frac{E(r,\theta,\phi)}{E_{\max}}$$

$$E_n(\theta,\phi)(dB) = 20\log\left[E_n(\theta,\phi)\right]$$

It is interesting to note that a normalized electric field pattern in dB will be identical to the power pattern in dB.

$$P_n(\theta,\phi)(dB) = 10\log\left[P_n(\theta,\phi)\right]$$

A polar plot

(a)

A rectangular plot

(b)
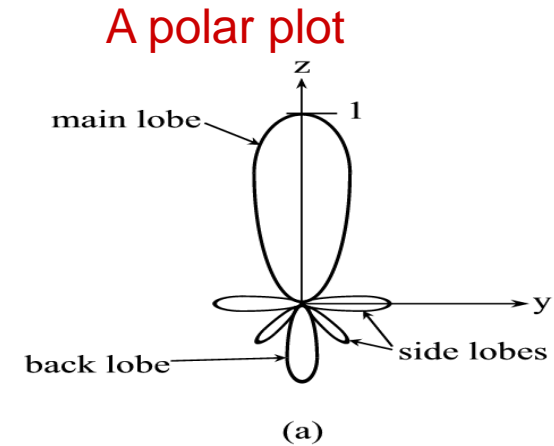
Antennas – Radiation Patterns

Radiation Pattern:

It is clear in Figure that in some very specific directions there are zeros, or nulls, in the pattern indicating no radiation.

The protuberances between the nulls are referred to as lobes, and the main, or major, lobe is in the direction of maximum radiation.

There are also side lobes and back lobes. These other lobes divert power away from the main beam and are desired as small as possible.

Beam Width:

One measure of a beam's directional nature is the *beamwidth*, also called the half-power beamwidth or 3-dB beamwidth.

A polar plot

main lobe
back lobe
side lobes
(a)

A rectangular plot

0 dB
-3 dB
beamwidth
(b)
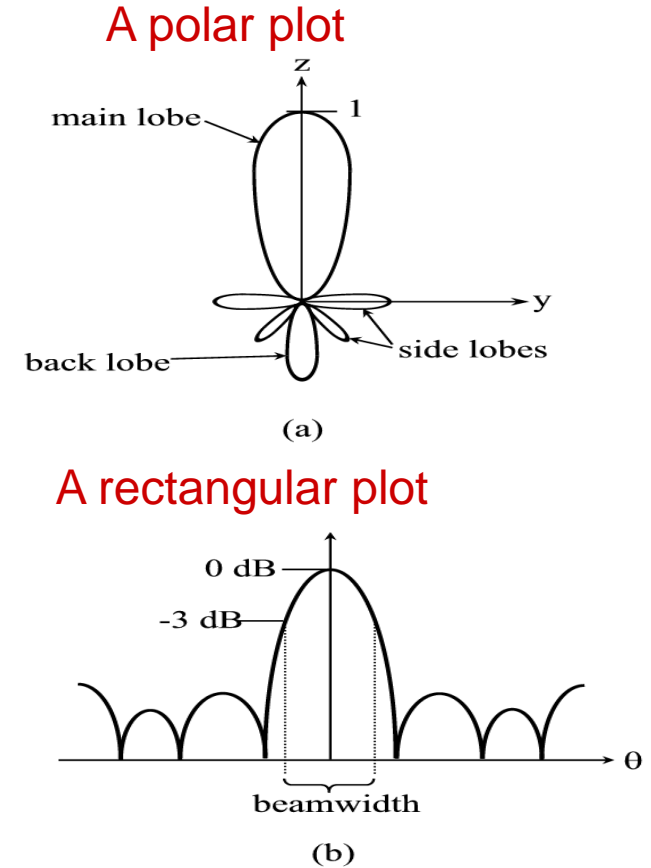
Antennas – S

Antenna Pattern Solid Angle:

A *differential solid angle*, $d\Omega$, in sr, is defined as

$$d\Omega = \sin\theta\, d\theta\, d\phi.$$

For a sphere, the solid angle is found by integrating

$$\Omega = \int\limits_{\phi=0}^{2\pi}\int\limits_{\theta=0}^{\pi} \sin\theta\, d\theta\, d\phi = 4\pi\,(sr).$$

An antenna's pattern solid angle,

$$\boxed{\Omega_p = \iint P_n(\theta,\phi)\, d\Omega}$$

All of the radiation emitted by the antenna is concentrated in a cone of solid angle Ω_p over which the radiation is constant and equal to the antenna's maximum radiation value.

(a)

(b)
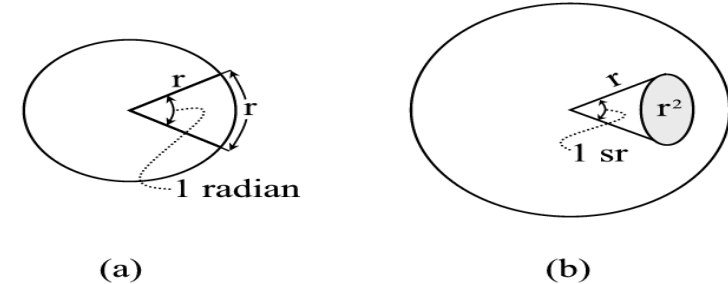
A radian is defined with the aid of Figure a). It is the angle subtended by an arc along the perimeter of the circle with length equal to the radius.
A steradian may be defined using Figure (b). Here, one steradian (sr) is subtended by an area r2 at the surface of a sphere of radius r.

Radiation pattern

Radiation pattern – variation of the field intensity of an antenna as an angular function with respect to the axis
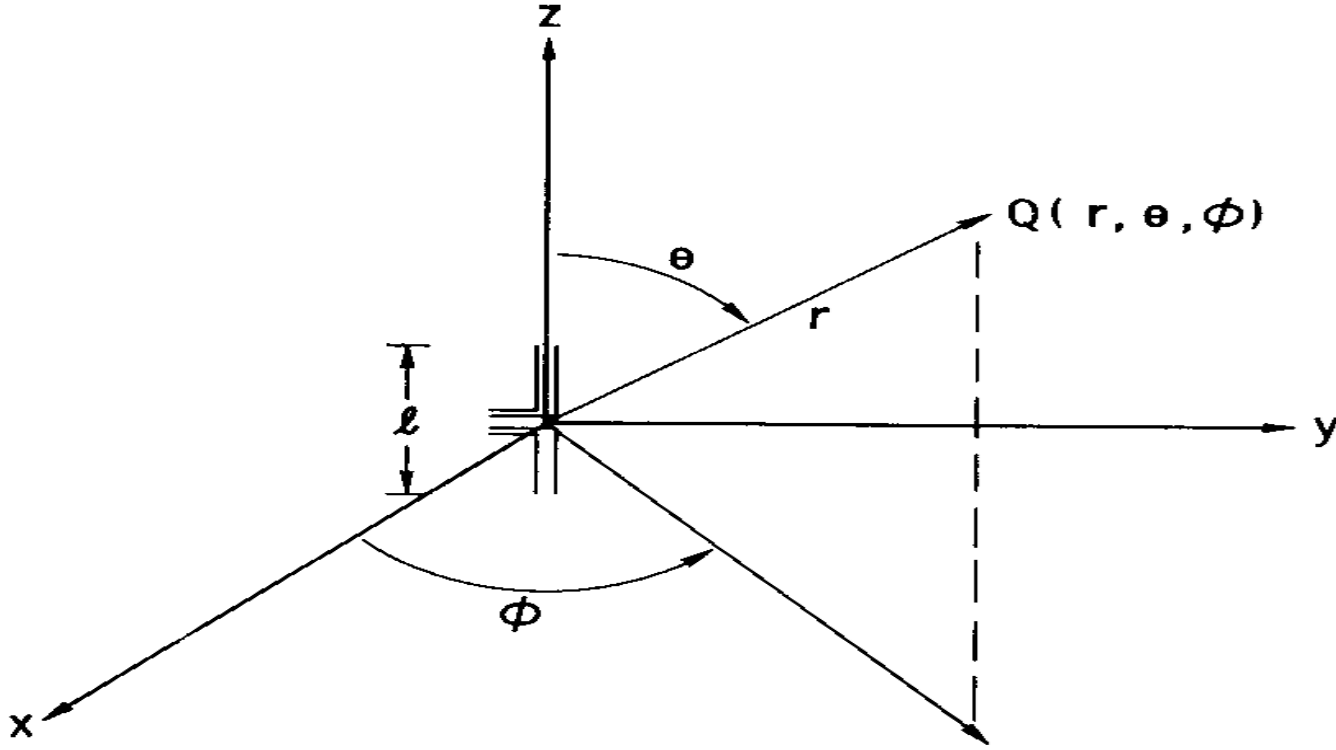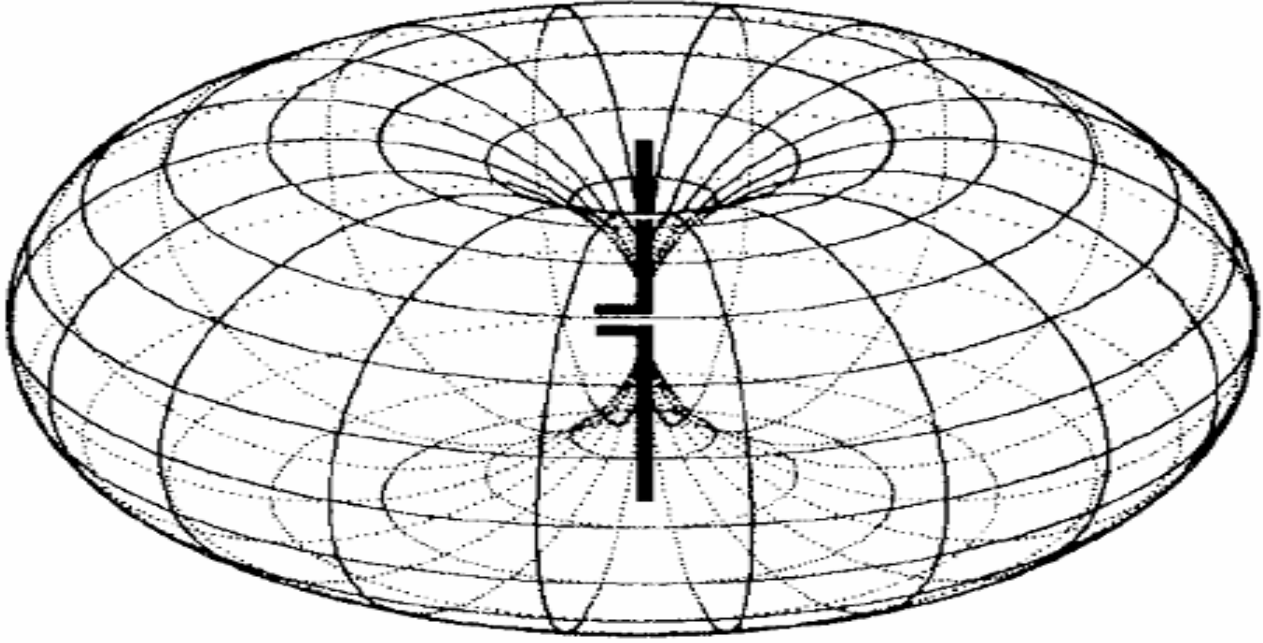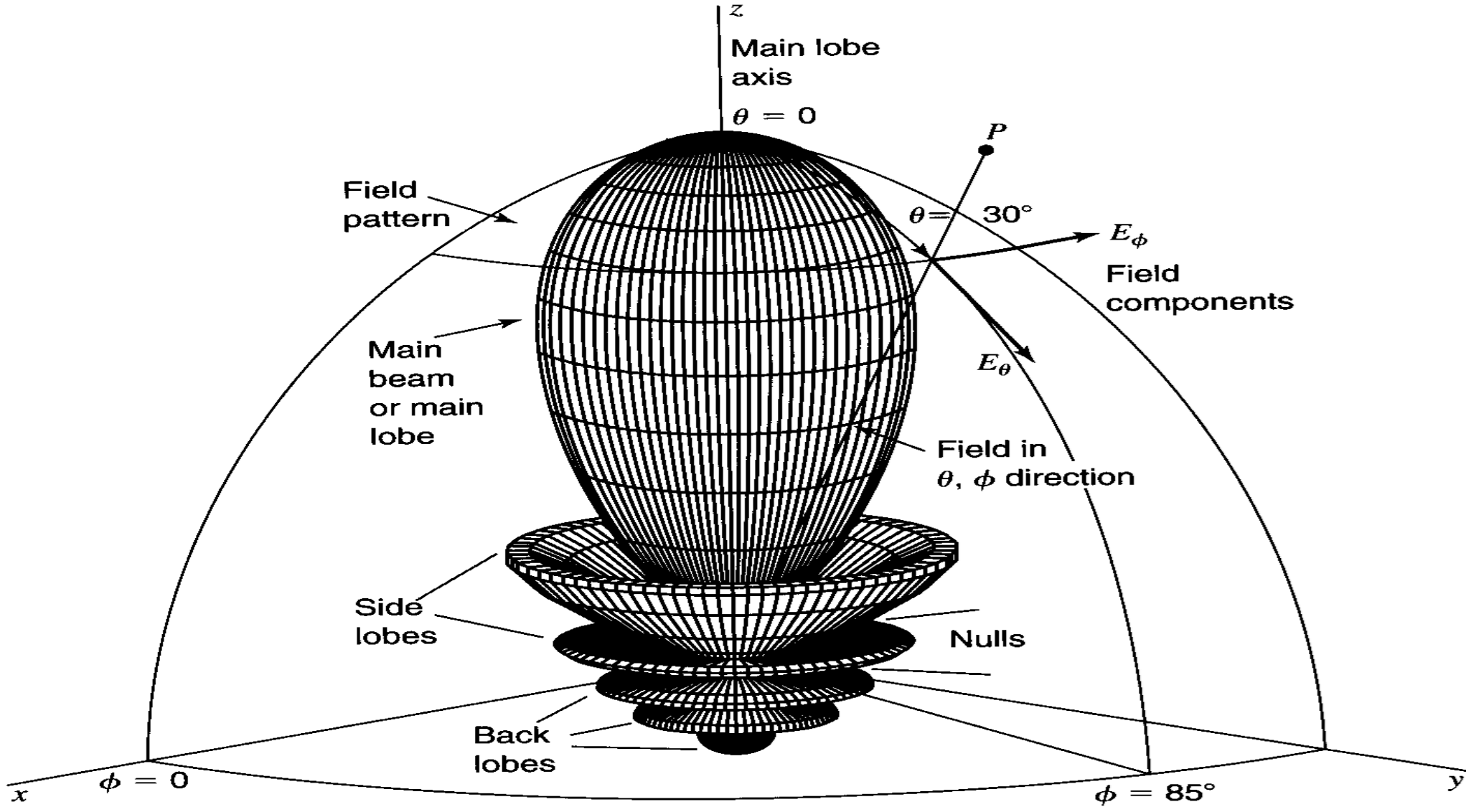

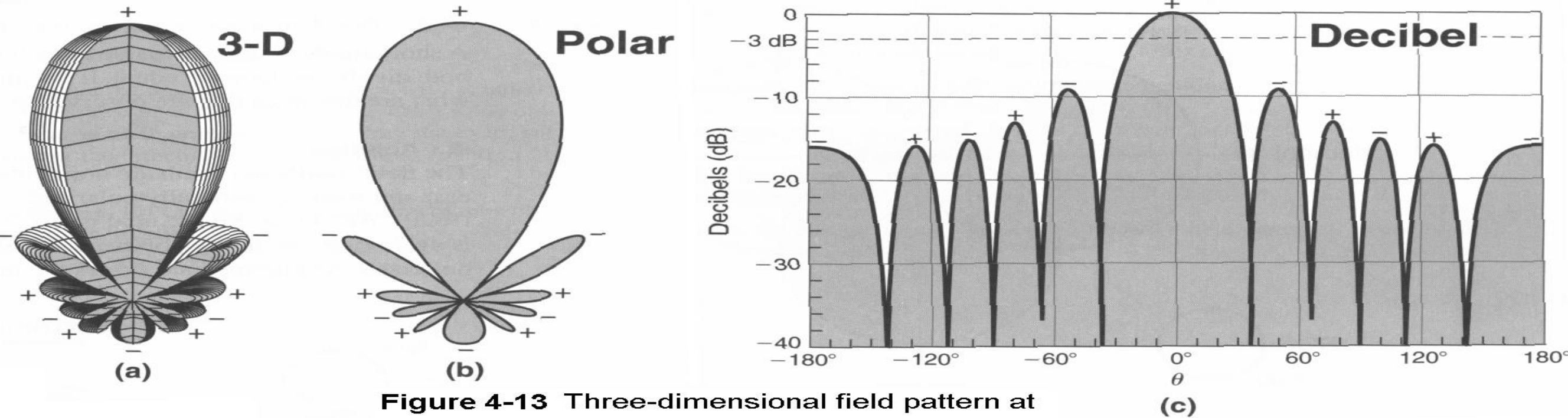Fig. 3.7   Short dipole placed at the origin of a spherical coordinate system.

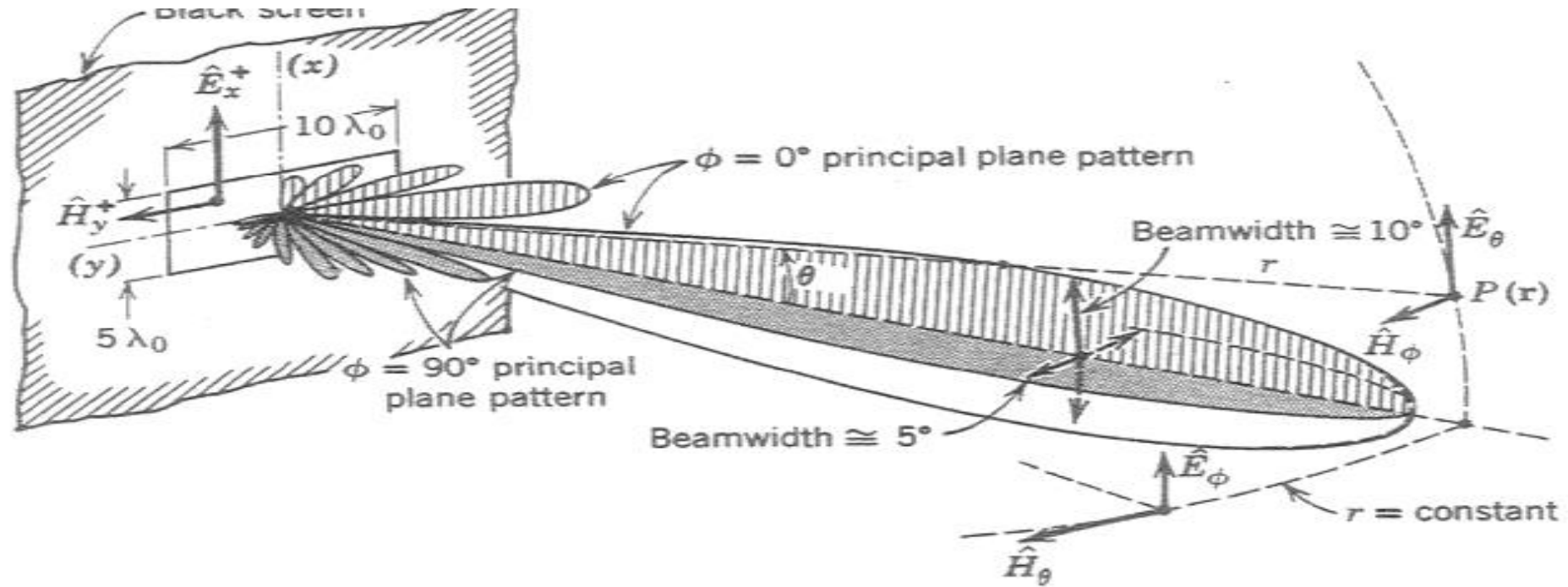Three-dimensional representation of the radiation pattern of a dipole antenna
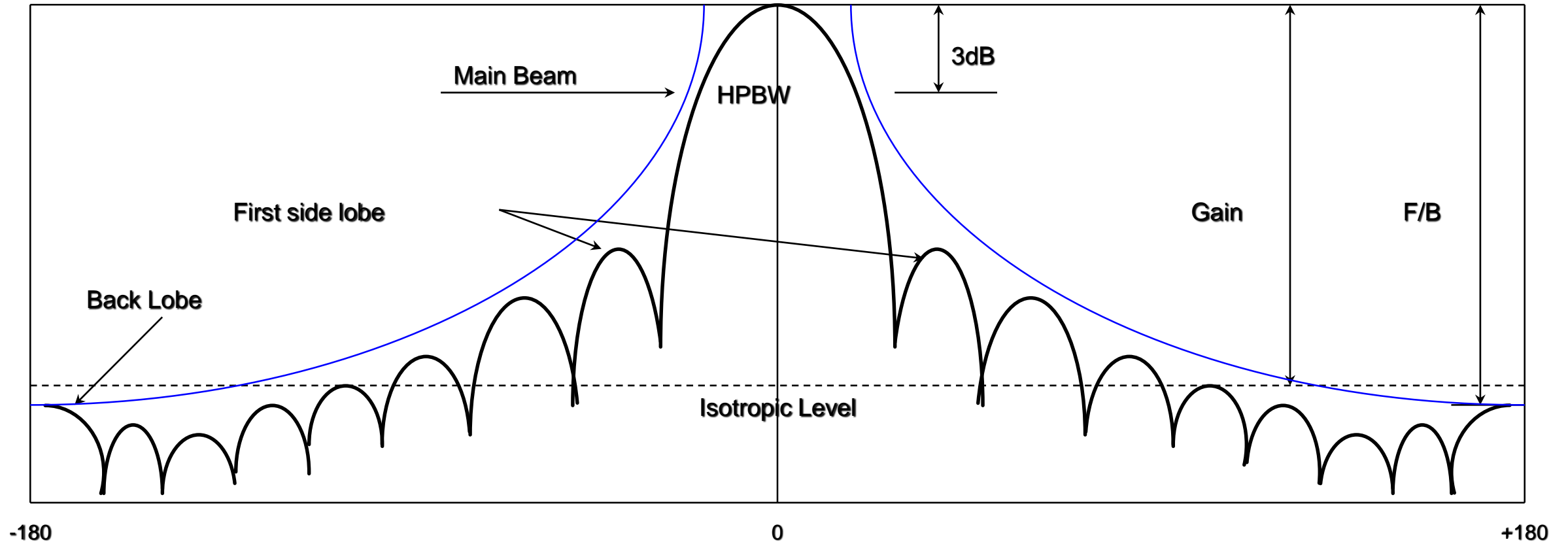
80

Radiation pattern

Radiation pattern

Figure 4-13 Three-dimensional field pattern at (a), polar pattern at (b), and decibel pattern at (c) showing alternate phasing (+ and -) of pattern lobes.

Antenna Pattern 3D

Antenna Radiation Pattern
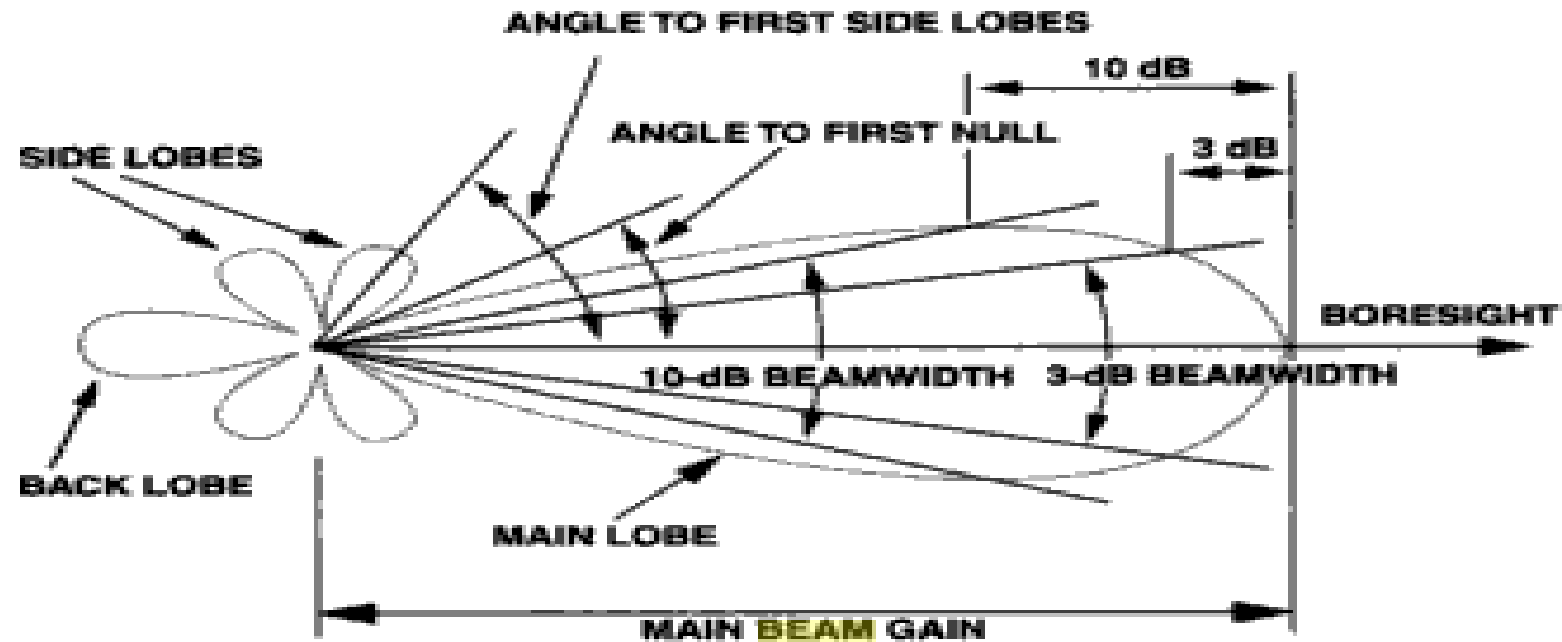(Cartesian Representation)

# Antenna beam definitions

Figure 3.1 Antenna parameter definitions are based on the geometry of the antenna gain pattern.
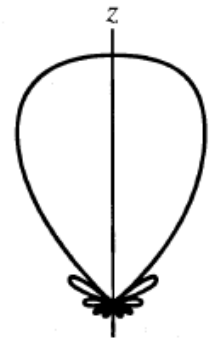
Antenna Pattern Types

- **Omnidirectional** – radiation response is constant in *one* of the principal planes of the antenna.

- **Isotropic** – antenna radiates equally in *all* directions in 3D space; theoretically impossible to realize, but a useful reference for quantifying how directive real antennas are.

- **Broadside** – main beam is normal to the plane or axis containing the antenna. An example for an antenna oriented along the z-axis is shown in Figure

- **Endfire** – main beam is *in the plane* or parallel to the axis containing the antenna. An example for an antenna oriented along the z-axis is shown in Figure
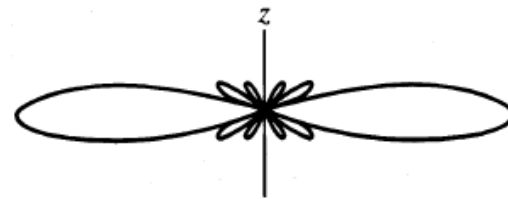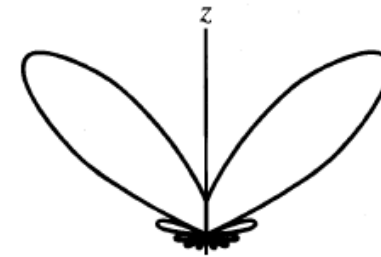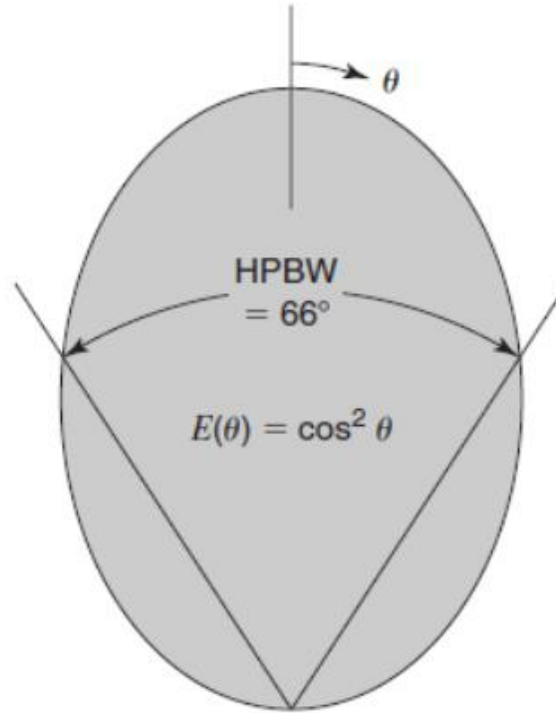

(a) Broadside   (b) Intermediate   (c) Endfire

Half-Power Beamwidth

An antenna has a field pattern given by

$$E(\theta) = \cos^2 \theta \quad \text{for } 0° \le \theta \le 90°$$

Find the half-power beamwidth (HPBW).
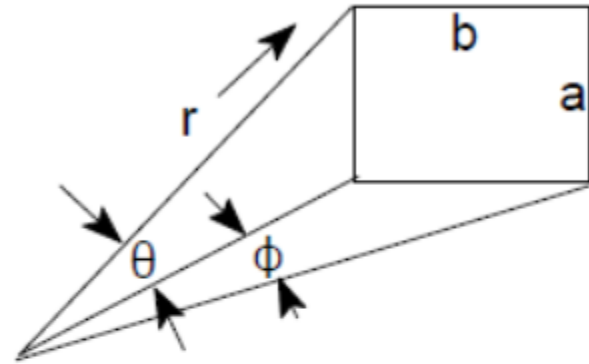

HPBW = 66°

$E(\theta) = \cos^2 \theta$

◼ Solution

$E(\theta)$ at half power = 0.707. Thus $0.707 = \cos^2 \theta$ so $\cos \theta = \sqrt{0.707}$ and $\theta = 33°$

$$\text{HPBW} = 2\theta = 66°$$

Pattern & Gain

Approximating the antenna pattern as a **rectangular** area:

$$a = r \sin \theta, \quad b = r \sin \phi, \quad area = ab = r^2 \sin \theta \sin \phi$$

$$G = \frac{Area\ of\ Sphere}{Area\ of\ Antenna\ pattern} = \frac{4\pi r^2}{r^2 \sin\theta\ \sin\phi} = \frac{4\pi}{\sin\theta\ \sin\phi}$$

Where $\theta = BW_\theta$, and $\phi = BW_\phi$
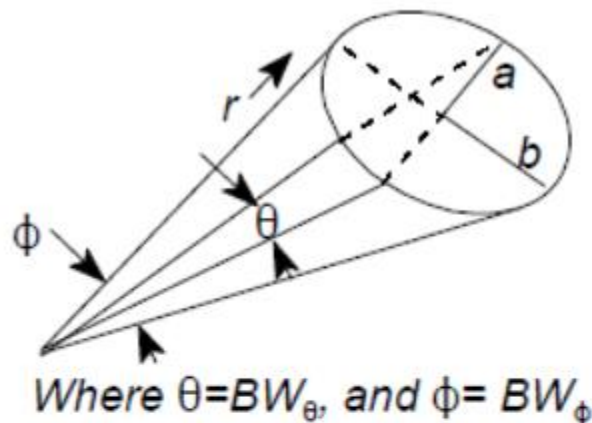
For small angles, $\sin \phi = \phi$ in radians, so:

$$G = \frac{4\pi}{\sin\phi\ \sin\theta} = \frac{4\pi}{\phi\ \theta\ (radians)} = \frac{4\pi}{\phi\ \theta}\left(\frac{360°}{2\pi}\frac{360°}{2\pi}\right) = \frac{41253}{\phi\ \theta\ (degrees)} \quad or \quad \frac{41253}{BW_\phi\ BW_\theta\ (degrees)}$$

The second term in the equation above is identical to equation [3].

Converting to dB, $G_{max}(dB) = 10\ Log\left[\dfrac{41253}{BW_\phi\ BW_\theta}\right]$ with BW_ϕ and BW_θ in degrees

Approximating the antenna pattern as an **elliptical** area:

Area of ellipse $= \pi\, a\, b = \pi[\ (r \sin \theta)/2\ ][\ (r \sin \phi)/2\ ] = (\pi\, r^2 \sin \theta \sin \phi)/4$

Where $\theta = BW_\theta$, and $\phi = BW_\phi$

$$G = \frac{Area\ of\ Sphere}{Area\ of\ Antenna\ pattern} = (4\,\pi r^2)\left(\frac{4}{\pi\, r^2 \sin\theta\ \sin\phi}\right) = \frac{16}{\sin\theta\ \sin\phi}$$

For small angles, $\sin \phi = \phi$ in radians, so:

$$G = \frac{16}{\sin\phi\ \sin\theta} \approx \frac{16}{\phi\ \theta\ (radians)} = \frac{16}{\phi\ \theta}\left(\frac{360°}{2\pi}\ \frac{360°}{2\pi}\right) = \frac{52525}{\phi\ \theta\ (degrees)} \quad or \quad \frac{52525}{BW_\phi\ BW_\theta\ (degrees)}$$

For a very directional radar dish with a beamwidth of 1° and an average efficiency of 55%:

Ideally: G = 52525, or in dB form: 10 log G = 10 log 52525 = 47.2 dB

With efficiency taken into account, G = 0.55(52525) = 28888, or in log form: 10 log G = 44.6 dB

Kaynaklar

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- https://cdn.rohde-schwarz.com/pws/dl_downloads/premiumdownloads/premium_dl_brochures_and_datasheets/premium_dl_whitepaper/Antenna_Basics_8GE01_1e.pdf

- Lecture notes from internet.

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Sincerely,

Dr. Cahit Karakuş

cahitkarakus@gmail.com

thank you